

Prediction of Mean Overtopping Discharge at Vertical Seawalls Using MLR and GLM Statistical Approaches

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ABSTRACT

Wave overtopping at breakwaters is one of their essential hydraulic characteristics when determining the design crest level. This study concentrates on developing a new practical formula on predicting wave overtopping, by implementing two different statistical models, Multiple Linear Regression model (MLR) and Generalized Linear Regression model (GLM). The models consider dependency of overtopping on a wide variety of quantities and yield to simple forms of prediction. Such statistical analysis are performed on a set of data called CLASH (Crest Level Assessment of Coastal Structures by full scale monitoring, neural network prediction and Hazard Analysis on permissible wave overtopping); the most complete and available database on overtopping phenomena. Proposed equations are compared with most recently extracted as well as successful ones. Comprehensive assessments clearly show more accurate predictions in the case of mean overtopping at vertical seawalls.

1. Introduction

Hydraulic responses of seawalls have the most important rule on their design process. Such parameters are wave run-up and run-down, wave overtopping, wave reflection and wave transmission [1]. Among them, wave overtopping is estimated to determine the crown elevation of the structure. Overtopping occurs when the highest run-up level surpasses the structure crown, sometimes causing damage to the coastal infrastructures. In the design process, precise estimation of all aforementioned hydraulic responses is of great importance and among them the overtopping plays a unique role. When, one estimates it lower than its actual amount, in practice it might halt port operations. Inversely, by an overestimation of the parameter, the structural design of the seawall could become non-economic. So, a strong parameterization including all of the relevant factors is required to achieve a comprehensive estimation as tried by pioneers. Prediction of wave overtopping and its impact on vertical seawalls and breakwaters have been considered in some prior researches and different formulas have been obtained. Formula for structures with vertical walls like vertical

seawalls are available in Franco et al.[2-4], Allsop et al.[5,6] and Besley et al. [7]. It has been actually one of the most interesting subjects during the last years resulting in promising progresses in this area. European project CLASH [8], have had an inevitable rule in such developments. CLASH provides a complete record of small-scale as well as full-scale data encouraging researchers to investigate overtopping at breakwaters more carefully, using different assessment tools. For example, there is a precious effort to present a model by means of artificial neural network, [9]. However, EurOtop [10] as the most successful try have mentioned that their model shows that, 68% of predictions lie within the desirable range of data. But, the complexity of the previous empirical equations is less than the aforementioned artificial neural network model. Although this simplicity is not a significant advantage of empirical formulas, but this might facilitate the first estimation of overtopping discharge in coastal structures design. Thus, this study is conducted in a similar way.

In section 2, a brief historical background on empirical formulas to estimate the overtopping

discharge at vertical structures is presented. In section 3, the CLASH overtopping database for vertical seawalls is briefly introduced. Section 4 represents the dimensionless parameters related to overtopping and their associated models. In section 5, the statistical models, used in this study; i.e., MLR and GLM are shortly described and the methodology and the results of model estimation are also represented in this section. Here, MLR and GLM are fitted on dimensionless parameters related to overtopping which were used in prior literatures as well as a new dimensionless parameter introduced in this study. For mentioned parameters their inner-relations are scrutinized and finally two new formulas are attained. Comparisons between different models are explained in section 6. Performance of each new formula is compared with that of the old and well-known competitor EurOtop in the case of vertical seawalls. Strong statistical accuracy assessments show much more coverage of CLASH database for our new MLR-based formula. Scatter diagram of overtopping discharges for each formula has been drawn to clarify the errors.

Table 1 lists all main dimensional parameters used in this research.

Table 1. Dimensional parameters related to the study

Parameter	Unit	Description
$H_{m0toe}=H_s=4(m_0)^{0.5}$	[m]	Wave Height
T_m	[s]	Average wave period
T_p	[s]	Wave period corresponding to the peak of the wave spectrum
$T_{0m}=(m_0/m_2)^{0.5}$	[s]	Mean wave period of the wave spectrum in deep water
T_{op}	[s]	Wave period associated with the spectral peak in deep water
L_{op}	[m]	Wave length associated with the spectral peak period in deep water
S_{op}	[-]	Wave steepness
q	[m ³ /s/m]	Overtopping discharge
R_c	[m]	Crest freeboard
$h_t=h_s=d$	[m]	Water depth at the toe of the structure

2. Historical background on overtopping at vertical structures

Since 1950's, several overtopping models have been developed considering mean overtopping discharge q , which is the flow rate per meter run (m³/s/m or l/s/m). Use of the 'mean' overtopping discharge is because q is a 'stable' parameter over several waves comparing to an individual overtopping wave volume. The most important group of models to predict overtopping is empirical one. The group is generally constructed using regression models based on overtopping data, obtained from physical modeling. Such models are constituted of a relationship between a dimensionless discharge and one or more dimensionless parameter,

for instance the dimensionless crest freeboard. Here, they are briefly introduced.

Ahrens et al. [11] conducted tests on some structures like seawalls, revetments, sloping structures, composite structures and vertical walls. Eq.(1) is their proposed model for overtopping which covers the vertical walls:

$$\frac{q}{\sqrt{gH_s^3}} = a \times \exp \left[-b \times \left(\frac{R_c}{(H_s^2 L_{op})^{1/3}} \right) - c \times S_{op}^{0.5} \right] \quad (1)$$

Franco et al. [2] considered several series of model tests on traditional vertical-face caissons, perforated ones, caissons with shifted sloping parapets and composite structure types, in order to study the overtopping response of mentioned structures. Eq.(2) shows their overtopping model which has been proposed for relatively deep water condition; within the range of $0.9 < R_c/H_s < 2.3$.

$$\frac{q}{\sqrt{gH_s^3}} = 0.2 \times \exp \left[-4.3 \times \frac{R_c}{H_s} \cdot \frac{1}{\gamma} \right] \quad (2)$$

In Eq.(2), γ is the reduction factor which depends on the specific structure geometry.

Allsop et al. [5] found that, Franco et al. model has a defect that, for larger values of R_c/H_s , it underestimates overtopping discharge. They proposed a similar model as Franco et al. [2] but with new improved coefficients, valid within a wider range, i.e. $0.03 < R_c/H_s < 3.2$.

$$\frac{q}{\sqrt{gH_s^3}} = 0.03 \times \exp \left[-2.05 \times \frac{R_c}{H_s} \right] \quad (3)$$

This model could be used in deep water condition as well as in shallow water. Their further research showed that the type of incident wave condition influences the overtopping performance of vertical walls. They have defined a wave parameter, h^* as follow:

$$h^* = (h_t/H_s) \cdot (2\pi h_t/gT_m^2) \quad (4)$$

In order to consider the difference between reflecting ($h^* > 0.3$) and impacting waves ($h^* \leq 0.3$):

$$\frac{q}{\sqrt{gH_s^3}} = 0.05 \times \exp \left[-2.78 \times \frac{R_c}{H_s} \right] \quad \text{for } h^* > 0.3 \quad (5)$$

$$\frac{q}{\sqrt{gh_t^3 h^{*2}}} = 1.37 \times 10^{-4} \left(\frac{R_c}{H_s} \times h^* \right)^{-3.24} \quad \text{for } h^* < 0.3 \quad (6)$$

Besley et al. [7] showed that, not only the values of H_s and T_p affect the mean overtopping discharge, but the form of incident wave breaking on the structure, has an important rule. If waves, relative to depth, at the

structure wall are large, they may break directly onto the structure, causing significantly more unexpected overtopping. According to indiscrete data from model tests in the UK [12] and the Netherlands [13], Eq.(7) has been driven as follow:

$$\frac{q}{h_*^2 \sqrt{gh_s^3}} = 0.05 \exp\left(-2.78 \frac{R_c}{H_s}\right) \quad (7)$$

Which is valid for $0.03 < R_c/H_s < 3.2$.

Franco et al. [4] based on the re-analysis of their prior data and also the results of 3D model studies performed at DH¹ [14], presented their new equation as:

$$\frac{q}{\sqrt{gH_s^3}} = 0.082 \times \exp\left[-3 \times \frac{R_c}{H_s} \cdot \frac{1}{\gamma}\right] \quad (8)$$

Their new model is extended for the effect of oblique and short-crested waves. In Eq.(8), γ considers such effects by calculating two parameters, i.e. γ_β and γ_s , presenting eight possible conditions.

EurOtop gives guidance on analysis and/or prediction of wave overtopping by wave action. In this manual, guidance for the assessment of overtopping at vertical coastal structures, for instance, commonly used set of equations to estimate the wave overtopping of caisson breakwaters, are presented. The manual itself suggests a set for three probabilistic design equations for three main conditions as follows:

For non-breaking waves ($h_* > 0.3$):

$$\frac{q}{\sqrt{gH_{mo}^3}} = 0.04 \exp(-2.6 \frac{R_c}{H_{mo}}), 0.1 < \frac{R_c}{H_{mo}} < 3.5 \quad (9)$$

For breaking waves ($h_* \leq 0.2$):

$$\frac{q}{h_*^2 \sqrt{gh_s^3}} = 1.5 \times 10^{-4} \left(h_* \cdot \frac{R_c}{H_{mo}}\right)^{-3.1}, 0.03 < h_* \cdot \frac{R_c}{H_{mo}} < 1 \quad (10)$$

And, for broken waves ($h_* \leq 0.2$):

$$\frac{q}{h_*^2 \sqrt{gh_s^3}} = 2.7 \times 10^{-4} \left(h_* \cdot \frac{R_c}{H_{mo}}\right)^{-2.7}, h_* \cdot \frac{R_c}{H_{mo}} < 0.02 \quad (11)$$

In above equations, h_* is defined as:

$$h_* = \left(\frac{h_s}{H_{m0}}\right) \cdot \left(\frac{2\pi h_s}{gT_m^2}\right) \quad (12)$$

3. Wave overtopping database

During last three decades, large numbers of data sets on overtopping phenomena have been produced and used at universities and research institutes all over the world. Among them, CLASH consists of a composition of much of data as possible. CLASH data

is gathered within the CLASH project, originated from CLASH partners (80% of data) and also from non-CLASH institutes in and outside Europe (20% of data).

Over 10000 tests were carried out during the 2 phases of the set-up of the CLASH database. Note that during the second phase; wherein 4000 overtopping tests were conducted; the data base improved to final CLASH database by adding some parameters. Each overtopping test consists of 31 parameters (Eleven hydraulic parameters related to the wave characteristics and seventeen structural parameters as well as three general parameters).

Verhaeghe et al. [15] and Steendam et al. [16] gave a detailed account of overtopping database set-up. Detailed information about CLASH database provided by Verhaeghe [17]. In Van der Meer et al. [8], the final CLASH database on overtopping is publicly available.

In this study the CLASH database with 10000 data is used for modeling and the comparisons for the new presented models are performed with EurOtop set of equations.

4. Dimensionless parameters

In this section, different dimensionless parameters, related to overtopping phenomena, are introduced. These parameters are the initial parameters for our MLR [18] and GLM [19] analysis. Generally, there are two dimensionless forms of mathematical models (Eq.(13), Eq.(14)), which are used to explain overtopping according to the prior literatures [20]:

$$Q = a \cdot \exp(-b \cdot R) \cdot \exp(\varepsilon) \quad (13)$$

$$Q = a \cdot R^{-b} \cdot \varepsilon \quad (14)$$

In Eq.(16) and Eq.(17), Q and R , are dimensionless average discharge per meter and dimensionless freeboard, respectively. Table 2 gives an overview of the dimensionless parameters used in recent overtopping formulas along with the associated definitions for dimensionless discharge and freeboard. According to prior researches; explained in section 2; generally one has six different dimensionless parameters to find an appropriate relationship between mean overtopping discharge and different sea state and structural parameters. These parameters have been taken into account in this analysis. In each set of analysis either for GLM or MLR, one of the three parameters related to mean discharge (Q_i) and one or more parameter related to freeboard (R_i) used to find the best correlation between these two categories.

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Table 2. Dimensionless parameters used in analysis

Independent Variables	$R_1 = \frac{R_c}{H_s}$	$R_2 = \frac{R_c}{\sqrt[3]{H_s^2 L_{op}}}$	$R_3 = \left(\frac{h_i}{H_{m0}}\right) \cdot \left(\frac{2\pi h_i}{g T_{om}^2}\right)$
Dependent variables	$Q_1 = \frac{q}{\sqrt{g H_s^3}}$	$Q_2 = \frac{q}{g H_s T_{om}}$	$Q_3 = \frac{q T_{om}}{L_{om}^2}$

Here, a new dimensionless freeboard, R_4 , which is obtained from the combination of two simple dimensionless parameters, i.e. R_c/H_{m0} and R_c/L_{om} , is defined as Eq.(15):

$$R_4 = k \cdot \frac{R_c}{H_{m0}} \cdot \frac{R_c}{L_{om}} = \frac{R_c^2}{g H_{m0} T_{om}^2} \left(L_{om} = \frac{g T_{om}^2}{2\pi}, k = \frac{1}{2\pi} \right) \quad (15)$$

This new parameter, which is easily obtained, is employed as a new independent parameter in analysis. It also should be noted that, further investigations are possible by making other new parameters by combining the introduced parameters in Table 2 and Eq.(15) together; but according to appropriate results from the parameters included in this study, there is no need to search for other new parameters.

5. Model fitting

5.1. General

It is usual that statistical models are chosen according to the nature of the data. However because of simplicity, the first model which is usually picked up for the estimations is the linear model. This fitting is following by some lack of fit test like normality of response variable, co-linearity of dependent variable and variance homogeneity [21].

The whole equation of MLR is as follow:

$$y = \beta_0 + \sum_i^n \beta_i x_i + \varepsilon_i \quad ; i = 1, \dots, k \quad (16)$$

Where y is the dependent variable; x_1, x_2, \dots, x_n represent different independent variables, β_0 is the intercept, $\beta_1, \beta_2, \dots, \beta_n$ represent the corresponding regression coefficients that are defined as partial regression coefficients and ε_i is residual.

It is clear that an iterative process is needed to find the best model; and in each step, the indexes of lack of fit should be improved. The usual encountering problem in this process is that the distribution of dependent variable is not normal; and the solution is to use BOX-COX transformation [22].

In some situations it is claimed that the distribution of the dependent variable in the population-not in the sample- is not normal. In the other words, the distribution is intrinsic non-normal. As a result, there is not homogeneity in the variance of the data. In this situation using GLM is proposed. The whole equation of this model is as Eq.(17).

$$y = g(\beta_0 + \beta_1 x_1 + \dots + \beta_{p-1} x_{p-1}) + e \quad (17)$$

Wherein, e is residual and $g(\dots)$ is formally a function and the inverse function of $g(\dots)$, which is $f(\dots)$, is called the link function.

This link function plays the role of showing the non-linear relationship between dependent and independent variables and it is determined based on the distribution of dependent variable. As for the linear model, all the parameter estimating and lack of fit testing phases have a complicated process, so some iterating methods like Newton-Raphson and Fisher-Scoring are used in the estimations.

In this research, because it is not clear that the main distribution of Q_1 in the population is either normal or not, beside the linear model the GLM is also enlisted. However, it should be mentioned that, the model which is more simple, efficient and precise is probably the most practical one.

5.2. Applying MLR

One of the dependent variables among all has to be selected, by which, the fitted model results in acceptable outputs. To this aim, at first, normality of distribution of selected dependent variable is tested. Among all, Q_1 (Table 2) showed more acceptable correlation; but as Figure 1 shows, the distribution of Q_1 , as a dependent parameter, has not normal distribution. So the Box-Cox transformation should be used in order to normalize the Q_1 .

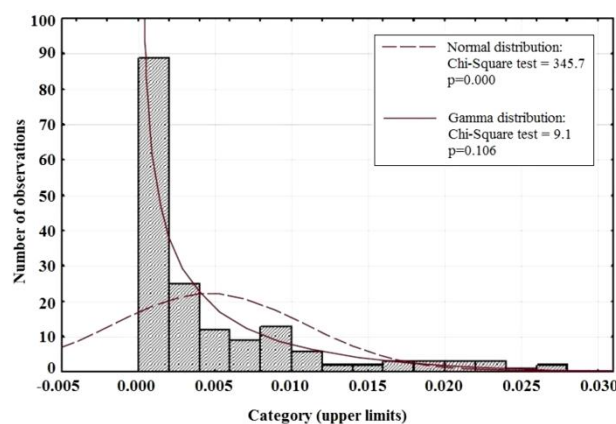


Figure 1. Test of normality of distribution and gamma probability density function of Q_1

Hence, the normalized form of Q_1 is obtained as Eq.(18):

$$\bar{Q}_1 = \frac{\left(\frac{q}{\sqrt{g H_{m0}^3}} \right)^{0.1416} - 1}{0.1416} \quad (18)$$

In order to detect the co linearity of variables, VIF² and Tolerance are calculated using Eq.(19):

$$VIF_i = \frac{1}{1 - R_i^2} \quad (19)$$

$$Tolerance = 1 - R_i^2 \quad (20)$$

² Variance Inflation Factor

Wherein, R_i^2 is the determination coefficient of regression equation.

It is obvious from Table 3 that variables R_2 and R_4 have co linearity with other variables. In comparison to R_2 , R_4 shows smaller values of VIF, so R_4 is applied in the model and R_2 is removed.

Table 3. The co linearity check for variables

Variable	Co linearity Statistics	
	Tolerance	VIF
$R_1 = R_c/H_s$	0.095	10.488
$R_2 = R_c/\sqrt[3]{H_s^2 L_{op}}$	0.025	40.663
$R_3 = (h_t/H_{m0}) \cdot (2\pi h_t / (gT_{om}^2))$	0.827	1.209
$R_4 = R_c^2 / (gH_{m0} T_{om}^2)$	0.055	18.067

Fitting different linear models to Q_1 , R_1 , R_3 and R_4 , the residual analysis and calculation and comparison of R^2 with stepwise methods, it is became obvious that the best model is as Eq.(21):

$$\frac{q}{\sqrt{gH_{m0}^3}} = 6.9 \cdot 10^{-5} \cdot \left[-0.785 \cdot \left(\frac{R_c}{H_{m0}} \right) + 6.743 \cdot \left(\frac{R_c^2}{gH_{m0} T_{om}^2} \right) + 0.024 \cdot \left(\frac{h_t}{H_{m0}} \right) \cdot \left(\frac{2\pi h_t}{gT_{om}^2} \right) + 2.484 \right]^{7.062} \quad (21)$$

By means of MLE³ method, the parameters related to the resulted equation from MLR are calculated and results are presented in Table 4. Based on experimental work limitations, Eq. (21) is valid for $0.1 < R_c/H_{m0} < 3.5$.

Table 4. Parameters prediction and MLR model coefficients

	β	Std.Err.	p-value
Intercept	-2.206	0.086	0.000000
R_1	-1.282	0.078	0.000000
R_3	-3.493	0.266	0.000000
R_4	16.673	5.043	0.001158

Std.Err.: Standard Error of parameter estimate

β : parameter estimate

P-value: the probability of obtaining a test statistic at least as extreme as the one that was actually observed, assuming that the null hypothesis is true

According to Table 4, estimated parameters are significant in 95% confidence (the p-value of all coefficients are less than %5). So, Eq.(21) is obtained as a new model in order to estimate the mean overtopping discharge at vertical seawalls.

This operation is conducted for two other dependent variables with all independent variables; but none of them showed better results comparing to Eq.(21).

5.3. Applying GLM

In previous section, it is shown that distribution of Q_1 is not normal. By means of K-S⁴ test for distribution of Q_1 , it is appeared that its distribution is gamma (the continuous line in Figure 1).

In this condition, GLM, as a regression model, can be used besides MLR by transforming dependent variable, the link function is in inverse form of GLM as Eq.(22).

$$y_i = \frac{1}{\alpha + \sum_{i=0}^{p-1} \beta_i x_i} + \varepsilon_i \quad (22)$$

By applying GLM with stepwise methods for all variables, the best relation is found between two variables Q_1 and R_1 , in the configuration of Eq.(23).

$$\frac{q}{\sqrt{gH_{m0}^3}} = 0.0058 \cdot \left(\frac{R_c}{H_{m0}} \right)^{-1} \quad (23)$$

As well as MLR modeling procedure in section 5.2, in this method, the process of examining output results and goodness of fit for different models built from two other dependent variables together with all independent variables (Table 2), is taken into account and Eq.(23) showed better results. Eq. (23) is also valid for $0.1 < R_c/H_{m0} < 3.5$.

6. Comparison between different models

In order to possible comparing two different introduced models together and also with the EurOtop model, some statistical indexes associated with relational data between 0 and 1, as Eq.(24), (25), (27) and (28) are taken into account.

$$R^2 = \frac{\left(n \cdot \sum_1^n q_{calc} \cdot q_{est} - \left(\sum_1^n q_{calc} \right) \cdot \left(\sum_1^n q_{est} \right) \right)^2}{\left(\sqrt{ n \cdot \sum_1^n q_{calc}^2 - \left(\sum_1^n q_{calc} \right)^2 } \right) \left(\sqrt{ n \cdot \sum_1^n q_{est}^2 - \left(\sum_1^n q_{est} \right)^2 } \right)} \quad (24)$$

$$\sigma_{XG} = \exp \left(\sqrt{ \frac{1}{n} \sum_{i=1}^n \left(\ln \frac{q_{est}}{q_{calc}} \right)^2 } - (\ln(X_G))^2 \right) \quad (25)$$

Wherein, X_G is:

$$X_G = \exp \left(\frac{1}{n} \sum_{i=1}^n \ln \left(\frac{q_{est}}{q_{calc}} \right) \right) \quad (26)$$

$$BIAS = \frac{1}{n} \sum_{i=1}^n [\log(q_{est}) - \log(q_{calc})] \quad (27)$$

$$RMSE = \sqrt{ \frac{1}{n} \sum_{i=1}^n (\log(q_{est}) - \log(q_{calc}))^2 } \quad (28)$$

In Eq.(24) to (28), q_{calc} is the calculated value, q_{est} is the observation data and n is the total number of measurements.

³ Maximum Likelihood Estimation

⁴ Kolmogorov-Smirnov

A comparison between these two models as well as with EurOtop by means of the equations introduced before, is presented in Table 5.

Table 5. Results of statistical parameters for the different models

Statistical Index	MLR Model [Eq. (21)]	GLM Model [Eq. (23)]	EurOtop set of equations [Eq. (9), (10) and (11)]
R_2	87%	82%	68%
X_G	0.815	0.793	0.675
σ_{XG}	1.113	1.488	2.833
$BIAS$	-0.089	-0.121	-0.471
$RMSE$	0.207	0.407	0.483

According to Table 5, The MLR model equation has a higher level of performance in similar case while comparing with previous formula and the GLM model. Figure 2, Figure 3 and Figure 4 illustrate a comparison between observed and predicted mean overtopping discharge for EurOtop set of equations (Eq.(9), (10) and (11)), GLM model (Eq.(23)) and MLR model (Eq.(21)) respectively.

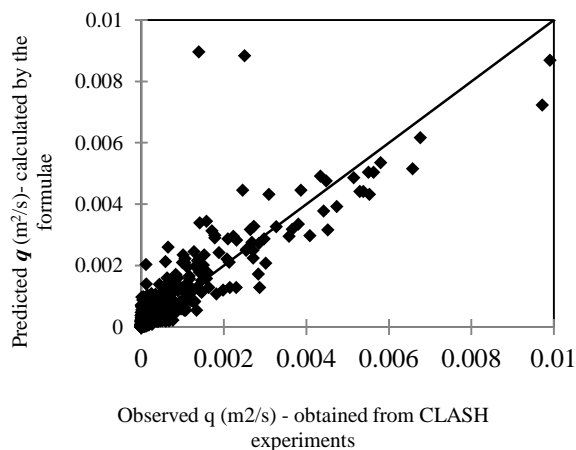


Figure 2. Mean overtopping discharge prediction, comparing observed and predicted for EurOtop set of equations, Eq.(9), (10) and (11)

This can be observed from Figure 2 that in some data points, EurOtop set of equations have high over prediction in their estimation.

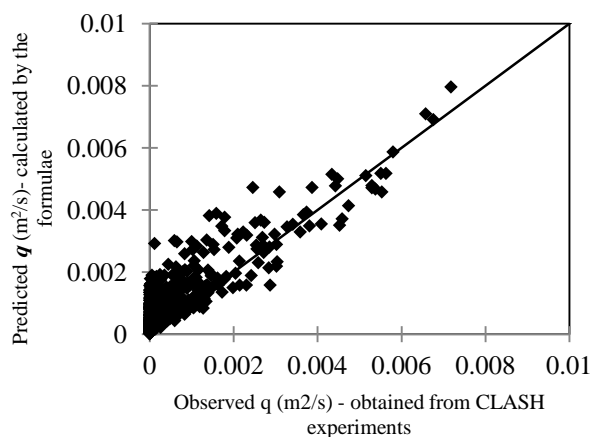


Figure 3. Mean overtopping discharge prediction; comparing observed and GLM predicted values (Eq.(23))

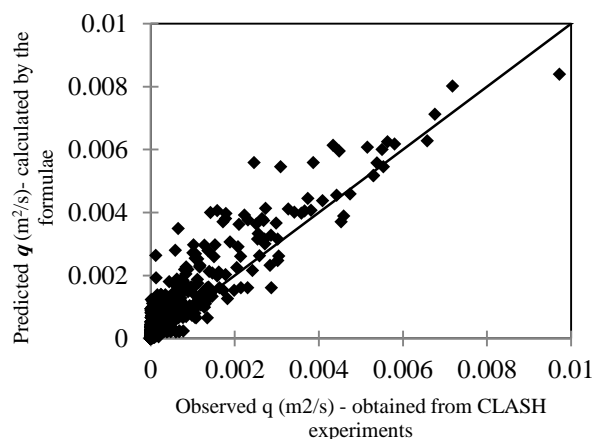


Figure 4. Mean overtopping discharge prediction; comparing observed and MLR predicted values (Eq.(21))

Figure 3 and Figure 4 show that, the new equations estimation (GLM and MLR models) has less scattering of data around the optimal trend line of discharge, comparing to that for EurOtop estimation.

7. Conclusions

Results of an empirical study on the seawalls overtopping data, carried out by CLASH, are represented conjointly with formulae (Eq.(21) and (23)) for estimating mean overtopping discharge at vertical seawalls. It is the most important hydraulic parameter for determining the design crest level of coastal structures, including the influence of wave height and period, crest freeboard of the structure and Water depth on the toe of the structure on the mean overtopping discharge. The proposed formulae has also been calibrated and confirmed against the formulae proposed by EurOtop (Eq.(9), (10) and (11)). The Eq.(23), which is driven by GLM model, has a very simple Structure and it is easy to be comprehended and according to table 5 its predictions seem acceptable. Presented model in Eq.(21), as the MLR model, in order to estimate the mean overtopping discharge of vertical seawalls, in comparison to the EurOtop model and also the new GLM model, shows more reliable results in statistical assessments and has just one rule comparing to three rules in EurOtop equations. The notability of the proposed formula is that it can estimate the dimensionless mean discharge well and with a good correlation in comparison with the most recently presented method in EurOtop.

Focus of the present research is on giving new formula which predict the mean overtopping discharge in vertical seawalls without foreshore with more accuracy than the previous ones. More researches in this area is recommended in order to develop more comprehensive formulas with better statistical indices.

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