A Practical Method for Investigation of Aerodynamic and Longitudinal Static Stability of Wing-in-Ground Effect

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ABSTRACT

The purpose of this paper is to present a fast, economical and practical method for mathematical modeling of aerodynamic characteristics of rectangular wing-in-ground effect (WIG). Reynolds averaged Navier-Stokes (RANS) equations were converted to Bernoulli equation by reasonable assumptions. Also Helmbold's equation was developed for calculation of the slope of wing lift coefficient in ground effect by defining equivalent aspect ratio (AR). Comparison of present work results against the experimental results has shown good agreement. Finally, according to the calculated aerodynamic coefficients, height static stability of WIG was evaluated by Irodov’s criterion in various ground clearance (h/c). A practical mathematical modeling with lower computational time and higher accuracy was presented for calculating aerodynamic characteristics of rectangular WIG. The relative error between the present work results and the experimental results was less than 8%. Also, the accuracy of the proposed method was checked by comparing with the numerical methods. The comparison showed fairly good accuracy. The evaluation of Irodov’s criterion shows that height static stability (HS) increases with reduction of the height. Aerodynamic surfaces in ground effect were used for reducing wetted surface and increasing speed in high-speed marine and novel aeronautical vehicles. The proposed method is useful for investigation of aerodynamic performance and HS of WIG vehicles and racing boats with aerodynamic surfaces in ground effect. The proposed method has reduced the computational time significantly as compared to numerical simulation that allows conceptual design of WIG craft to do with economical.

1. Introduction

The aerodynamic performance of airplanes and WIG crafts was increased in take-off and landing phase by the ground effect. Also, the aerodynamic lift of aerodynamically alleviated marine vehicle, due to using aerodynamic surfaces in ground effect, increases in near water surface and reduces wetted surface to achieve higher speed. When a wing moves near the ground or water surface, the aerodynamic lift of wing increases and wing-tip vortex does not able to propagate therefore induced drag reduces. Consequently, the lift to drag ratio of WIG increases. Many numerical and experimental studies performed to predict the aerodynamic characteristics of WIG. Fink and Lastinger (1961) presented experimental result of wing aerodynamic characteristics with low AR in close proximity of ground [1]. In this study, wing aerodynamic characteristics with Glenn Martin21 asymmetry airfoil section and AR=1, 2, 4 and 6 evaluated in different ground clearances. They used the two wings on opposite sides of the imaginary ground. All wings in ground effect enhanced the lift and reduced the drag which resulted in the increase of aerodynamic performance [1]. Kikuchi et al. (1997) studied numerical simulation of wing with NACA65A010 airfoil section in ground effect using the boundary element method. Their investigations were conducted to calculate the aerodynamic coefficient for long and short ground and validated results of numerical simulation with experimental result of wind tunnel. It was made clear that evaluation is possible with the pressure variation generated by the passing wing in accordance with the steady boundary element method on a long ground
plate and with the unsteady boundary element method on a short ground plate [2]. Rozhdestvensky (2000) researched about aerodynamic surface in extreme ground effect and he described the numerical approach of flow past aerodynamic surface on steady and unsteady conditions by using Laplace's equation and potential flow [3]. Steinbach (1997) commented on research of Hsiun and Chen that moving ground is better than fixed ground because air move on the ground before contact to airfoil and boundary layer produced then numerical result in ground effect was incorrected for ground clearance lower than 0.05 [4]. Park and Lee (2008) calculated aerodynamic characteristics of wing with Glenn Martin21 airfoil section and AR=1 in ground effect with and without endplate. They found that the endplate prevented the high pressure air from escaping out of the lower wing surface, reduced the influence of the wing-tip vortex [5]. Suh et al. (2011) experimental and numerical studies were performed to evaluate aerodynamic coefficients of WIG. The Vortex Lattice Method (VLM) simulated the wake deformation following the wing in the influence of the ground effect. Effect of design parameters such as angle of attack (AOA), aspect ratio (AR) and endplate were studies on the aerodynamic characteristics [6]. Kayiemet al. (2011) carried out aerodynamic characteristics of NACA4412 airfoil using FLUENT software and experimental measurements in low speed wind tunnel from -4° to 8° in ground effect [7]. Also Jung et al. (2012) investigated aerodynamic characteristics of NACA4406 airfoil using FLUENT software [8]. Luo and Chen (2012) presented experimentally investigation wing with NACA0015 airfoil in ground effect and shown pressure distribution around of airfoil and lift coefficient curve in different AOA and h/c [9]. Qu et al. (2014) carried out CFD simulation to study the flow field and aerodynamic properties of a NACA4412 airfoil in dynamic ground effect. The energy conservation equation for the air below the airfoil in dynamic ground effect can be used to calculation of relative pressure [10]. Maali et al. (2014) after calculation of aerodynamic coefficients of Wing by using theoretical method in ground effect developed a semi-empirical method for hydro-aerodynamic performance evaluation of an aerodynamically alleviated marine vehicle (AAMV) in take-off phase. Their practical method for determination of aerodynamic coefficients was well-validated against experimental data [11].

In the other hand, stability is very important aspect of designing for vehicles moving near the ground and sea surface. The stability characteristics play a great role in safety of WIG craft rather than aircraft due to cruised long time close proximity to the sea surface with high speed. Longitudinal stability can be divided into two sub division, static and dynamic modes. A WIG is statically stable if after disturbance, it has tendency to returns to the previous situation. Dynamic stability of WIG was usually satisfied if provide the static stability. In recent decades, several researches presented about longitudinal static stability of WIG. Irodov (1970) evaluated static stability of ekranoplan with aerodynamic coefficients [12]. Chun and Chang (2002) derived static and dynamic stability criteria for a 20 passenger WIG craft from the motion equations and wind tunnel test data [13]. Kornev and Matveev (2003) determined appropriate range of longitudinal static stability for stable flight in ground effect by using VLM [14]. Hendarko (2006) studied longitudinal static stability as well as flying qualities of small WIG using perturbation theory and semi-empirical method [15]. Kim et al. (2009) found the optimal position of the side wing attached on the WIG that can achieve the maximum lift and satisfy the height static stability criteria [16].

In this article was conducted practical approach for aerodynamic characteristics evaluation of rectangular WIG. Also Helmbold's equation was developed for calculation of the slope of wing lift coefficient in ground effect by definition equivalent aspect ratio (AR). Aerodynamic characteristics are lift coefficient, drag coefficient, pitching moment coefficient and lift to drag ratio. Results of present work for rectangular wing with AR=2 were compared to experimental results in h/c= 0.167 at AOA= 0° to 10° that good agreement were shown between the present work results and the experimental results. Also accuracy of the present work was evaluated respect to the numerical methods. Complete results were presented for the aerodynamic characteristics of rectangular wing in h/c=0.083, 0.167 and 0.333 at AOA between 0° to 10°. Finally, according to the calculated aerodynamic coefficients, height static stability of WIG with AR=2 in AOA=4° was evaluated by Irodov’s criteria in different h/c. The present work is useful for investigation of aerodynamic performance and static stability of WIG vehicle and racing boat with aerodynamic surface in ground effect.

2. Mathematical modeling
Lift coefficient, drag coefficient, pitching moment coefficient and lift to drag ratio are the most favorite WIG characteristics of aerodynamic. The h/c, AOA and AR have important effect on aerodynamic characteristics of WIG. According to over explanations, mathematical modeling in this study has two steps: 2-Dimensional (airfoil or infinite wing) and 2-Dimensional (finite wing) modeling.

2.1. 2-Dimensional modeling
WIG crafts and aerodynamically alleviated marine vehicles usually have speed between 200 to 300 km/h and Reynolds number more than $2 \times 10^5$ consequent boundary layer is thin [17].

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So flow field out of boundary layer around the wing can be considered irrotational. Mach number for WIG crafts and aerodynamically alleviated marine vehicles is less than 0.3 in maximum speed and the flow can be assumed incompressible. Also the fluid properties were taken to be constant and the effect of the viscous dissipation was assumed to be negligibly small. Mathematical modeling was done with below assumptions:

- Irrotational flow
- Invicisid fluid
- Incompressible flow
- Without energy loss
- Don’t occur stall

These assumptions convert RANS equations to Bernoulli equation. The Bernoulli equation can be applied for all points of flow field in out of boundary layer between the wing and ground. 2D modeling can be used for symmetric and asymmetric airfoil. In Figure 1 was shown sketch of the airfoil and coordinate system on it.

![Figure 1, Schematic diagram for the mathematical model of airfoil in ground effect](image)

In Figure 1, f(x) denotes the airfoil profile with respect to xoy coordinate system. f(x) can mark a symmetrical or asymmetrical airfoil. Using the relation \( h(x) = x\sin\alpha - |f(x)|/\cos\alpha \) is able to calculate the distance of each point at the lower surface of the airfoil from ground. Using the simplified governing equations between two points at the lower surface of the airfoil (points 1 and 2 in Figure 1), result in the relative pressure between these two points as follows:

\[
P(x) = \frac{\rho V_\infty^2}{2} \left[ 1 - \frac{h^2}{(h + h(x))^2} \right]
\]

(1)

The vertical force on airfoil was calculated by integrating from Eq. (1) as follow:

\[
F = \int_0^h \frac{\rho V_\infty^2}{2} \left[ 1 - \frac{h^2}{(h + h(x))^2} \right] dx
\]

(2)

Contribution of the lift coefficient \( (C_L') \) because of ground effect was determined from Eq. (2) and contribution of the lift coefficient \( (C_{LW}) \) in out of ground (free stream) was estimated by experimental result of other references. Consequently the lift coefficient of airfoil in ground effect by using superposition principle is equal to:

\[
C_{L_{WIG}} = C_{LW} + C_L'
\]

(3)

In fact, the distributed force acting on airfoil represented by concentrated force acting at the center of pressure. The concentrated lift force was shifted to quarter-chord point, namely, pitching moment. The pitching moment for asymmetry airfoil in ground effect is obtained from Eq.(2).

\[
M_{Airfoil} = \frac{\rho V_\infty^2}{2} \int_0^h \left[ 1 - \frac{h^2}{(h + h(x))^2} \right] x dx
\]

(4)

The pitching moment coefficient in ground effect is achieved similarly the lift coefficient by applying superposition principle as follows:

\[
C_{M_{WIG}} = C_{M_{W}} + C_M'
\]

(5)

where \( C_M' \) is contribution of the pitching moment coefficient due to enhancement of pressure in ground effect respect to free stream that obtain by Eq.(5) and \( C_{M_{W}} \) is contribution of the pitching moment coefficient due to pressure producing around of airfoil in free stream that estimate by experimental result of other references.

The drag coefficient is one of the important aerodynamic characteristics of wing. The drag coefficient of wing is a combination of three components [18]:

- Frictional drag coefficient \( (C_{D,f}) \): due to frictional share stress acting on the surface of the airfoil.
- Pressure drag coefficient \( (C_{D,p}) \): due to flow separation was caused by the imbalance of the pressure distribution in the drag direction on the airfoil surface
- Induced drag coefficient \( (C_{D,i}) \): a pressure drag due to the pressure imbalance in the drag direction caused by the induced flow (downwash flow) associated with the vortices created at the tips of finite wings

Then, drag coefficient of wing was written as below [18]:

\[
C_D = C_{D,f} + C_{D,p} + C_{D,i}
\]

(6)

Tow first term in right-hand of Eq.(6) are components of the drag coefficient in 2D (infinite wing or airfoil) and third term is component of the drag coefficient in 3D (finite wing). Affect of the ground effect was assumed negligible on boundary layer thickness and separation point location of airfoil. Then the friction and pressure drag coefficient don’t have different in
free stream with ground effect [5, 11]. Consequently
drag coefficient was not determined for airfoil in
ground effect. The frictional and the pressure drag
coefficient were obtained using experimental results
for 3D modeling of WIG. Calculation of the induced
drag coefficient was described in the next section.

2.2. 3-Dimensional modeling
In fact, airfoil is wing with infinite span that 2-D
analysis was applied for it. But 3-D analysis was used
for finite wing, because of the pressure difference
between the upper and lower wing surface was
produced wing-tip vortex and aerodynamic performance reduces by generating wing-tip vortex.
Consequently, aerodynamic performance of finite
wing is lower than airfoil. Anderson suggested an
equation for converting airfoil’s lift coefficient slope to
finite wing’s lift coefficient slope, namely Helmbold’s
equation [18].
But this equation is not applicable for converting the
airfoil’s lift coefficient slope to finite wing’s lift
coefficient slope in ground effect. Due to effect of
reduction of down wash and wing-tip vortex on the
lift generation in ground effect was not considered in
the Helmbold’s equation. On the other hand, the
Helmbold’s equation should be used in free stream.
Furthermore the Helmbold’s equation should develop
for calculation of the wing’s lift coefficient slope.
Wieselsberger (1992) presented an φ to consider the
ground effect on the prediction of the induced drag
coefficient by using Prandtl’s wing theory [19]. Using
this coefficient can be determining ARₑ in ground
effect as below:

\[
ARₑ = \frac{AR}{1 - \varphi}, \quad \varphi = \frac{1 - 1.32 \frac{h}{c}}{1.05 + 7.4 \frac{h}{c}}
\]

(7)
The influence coefficient (φ) is always smaller than 1
and consequently ARₑ in ground effect would be
larger than the actual AR out-of-ground effect. In fact,
effect of reduction of wing-tip vortex in ground effect
considered by decrease of the AR. Thus the
Helmbold’s equation changes to Eq.(8) in ground
effect with replacing the AR by the ARₑ:

\[
\frac{dC_L}{da}_{wing} = \frac{dC_L}{da}_{airfoil} \left[ \frac{1}{1 + \left( \frac{dC_L}{da}_{airfoil} \right)^2} \right] + \frac{dC_L}{da}_{airfoil} \frac{1}{\pi ARₑ}
\]

(8)
Where \( \frac{dC_L}{da}_{airfoil} \) and \( \frac{dC_L}{da}_{wing} \) are the airfoil’s lift
coefficient slope and the finite span wing’s lift
coefficient slope, respectively. Eq.(8) shows when

wing’s lift curve slope is lower than airfoil’s lift curve
slope and if AR is infinity, wing’s lift curve slope is
equal to airfoil’s lift curve slope.

After calculation of lift coefficient can be determined
induced drag. The contribution of induced drag is
more than contribution of frictional and pressure drag
on total drag.
The reduction of h/c can be lead to decrease of the
induced drag coefficient. Philips and Hunsaker (2013)
presented induced drag ground effect influence ratio
(σ) that closed form relation was used for estimation
induced drag coefficient in ground effect (Eq.(9))
[20].

\[
σ = 1 - \exp[-3.88 \frac{h}{b}^{0.66}]
\]

(9)
The power of exponential term of Eq.(9) by divided c
change to the dimensionless closed form relation that
appropriate relation for calculation induced drag
coefficient in ground effect as below:

\[
C_{D,i} = σ \frac{C_{i,0}}{e \pi AR}, \quad σ = 1 - \exp[-3.88 \frac{h}{c}^{0.66}]
\]

(10)
The σ is dependent on h/c and AR in Eq.(10) that
these are two important parameters on aerodynamic
performance of WIG. In Eq.(10), if h/c tend to
infinity, σ equal to 1 and also if h/c tend to infinity, σ
equal to 1.
The pitching moment was generated according to the
lift distribution around the wing. By converting the
infinite wing (airfoil) into a finite wing, both lift force
and pitching moment decrease. Consequently,
reduction in the pitching moment of the wing is
dependent on reduction in the value of wing lift with
respect to the airfoil lift. The pitching moment
coefficient of wing was determined according to
pitching moment coefficient of airfoil in ground effect
by using difference lift coefficient between wing and
airfoil.

3. Longitudinal static stability
Longitudinal static stability of the WIG is extremely
sensitive to both pitch and height variations. This is
different from stability of the conventional aircraft
where changes in height result in minimal force and
moment variations, and as such, was ignored and only
requires consideration for pitch stability. The lack of
the longitudinal stability in the WIG craft can lead to a
serious accident and major damages. A WIG is
statically stable if it has tendency to returns to the
previous situation after perturbation.
There are two neutral aerodynamic centers for WIG.
The aerodynamic center in pitch (xₑ) follows
conventional aerodynamics definition and is where the
moment acting on the configuration of WIG is
independent of the AOA. Similarly, the aerodynamic
centre in height (xₑ) was defined as the point where
moment is independent of the flying height. These
two aerodynamic centers can be obtained by considering the lift and moment coefficient curves with respect to AOA and h/c. Based on aerodynamic coefficients of WIG determined in the previous sections, longitudinal static stability of height can be mathematically expressed that was proposed by Irodov (1970) as follows [12]:

\[
HS = \frac{C_{Mz}}{C_{Lz}} = x_z - x_a \leq 0
\]

(11)

Where the second subscripts z and α indicate the derivative of the ground clearance and the angle of attack, respectively and HS means height stability. The WIG has static stability if, the aerodynamic centre in height was located upstream of the aerodynamic centre in pitch. In this study, coordinate system was considered on the leading edge of wing for calculation of the aerodynamic centers position. Kornev and Matveev (2003) have suggested upper and lower bound of suitable height static stability criterion in ground effect, \(-0.15 \leq H_S \leq -0.05\) [14].

4. Validation

In this section, results of present method were validated for the rectangular wing with Glenn Martin21 asymmetrical airfoil section and AR=2 in h/c=0.167 by experimental results of Fink and Lastinger, 1961. In Figure 2 was shown a comparison between the present work results and the experimental results for lift coefficient, drag coefficient and pitching moment coefficient of rectangular wing with AR=2 versus AOA in h/c=0.167.

5- Result and discussion

Corresponding performed validation and appropriation accuracy proposed approach; in this section examinations of aerodynamic characteristics of rectangular WIG were completely conducted. In Figure 3 was shown variations of AR_e respect to AR in various h/c. AR_e for WIG can be increased by decreasing the h/c.

In fact, Figure 3 depict that reduction of wing-tip vortex in ground effect can be considered by AR_e instead of AR that cause the increasing of wing aerodynamic performance. In Figure 4 was presented a comparison of the lift coefficient of wing with AR=1 between the experimental results and the present work results with numerical results that carried out by CFD [5] and VLM [21] in h/c= 0.167.
Figure 4, Comparison of the experimental result [1] and the present work result with numerical result [5, 21] for the lift coefficient of wing in h/c=0.167

The all of methods were shown in Figure 4 have good agreement with the experimental results. Maximum of errors for the present work result, numerical result by CFD method and VLM are respectively 9%, 7% and 14%. The numerical result by CFD method is closer than the present work result to the experimental result but the present work has computation time very shorter than CFD method.

Also in Figure 5 was shown a comparison of the drag coefficient of wing with AR=1 between the experimental results and the present work results with CFD results [5] in h/c=0.167.

Figure 5, Comparison of the experimental results [1] and the present work results with CFD results [5] for the drag coefficient of wing in h/c=0.167

The present work results and the numerical results (CFD) were shown in Figure 5 have appropriate agreement with the experimental results. Maximum of errors for the present work results and numerical results (CFD) are 16% and 15%, respectively. According to Figures 4 and 5, the present work results have good agreement with the experimental results and the numerical (CFD and VLM) results. These good agreements between results show that assumptions of present work are appropriate and reasonable.

The lift coefficient of wing with AR=2 versus various h/c in the AOA=0, 2, 4, 6 and 8 degree was depicted in Figure 6.

Figure 6, Lift coefficient for wing with AR=2 in ground effect

In Figure 6 was shown that the lift coefficient increases nonlinearly with reduction of h/c for wing with AR=2 and increase of lift coefficient in lower h/c is more than high h/c. The drag coefficient of wing with AR=2 versus various h/c in the AOA=0, 2, 4, 6 and 8 degree was presented in Figure 7.

Figure 7, Drag coefficient for wing with AR=2 in ground effect

In Figure 7 was presented the drag coefficient decreases with reduction of h/c and it increases with increase of AOA. The reduction of drag coefficient in ground effect is due to limited the wing-tip vortex. The increase of drag coefficient with increase of AOA is due to generation larger surface against of air flow and separation of air flow on the wing occurs sooner. The pitching moment coefficient of wing with AR=2 versus various h/c in the AOA=0, 2, 4, 6 and 8 degree was shown in Figure 8.

Figure 8, Pitching moment coefficient for wing with AR=2 in ground effect
According to Figure 8, variation of the pitching moment coefficient is nonlinear respect to h/c. The pitching moment coefficient increases with reduction of h/c (negative sign shows moment direction). Because of variations of the pitching moment coefficient is dependent to variation of the lift coefficient and the lift coefficient increases with decrease of h/c.

For h/c ≥ 0.4, the wing with Glenn Martin21 section is, indeed, completely out of ground effect and the significant changes of aerodynamic characteristics were not observed. However, for h/c < 0.3, because the wing begin to affect the ground effect, the aerodynamic characteristics change nonlinearly. Also for h/c < 0.1, the aerodynamic characteristics change extremely, due to the wing is very close to the ground.

The lift to drag ratio of wing with AR=2 in h/c=0.083, 0.167 and 0.333 was presented in Figure 9 according to the results of present work in Figures 6 and 7.

Corresponding Figure 9, reduction of h/c causes increase of the lift to drag ratio. Also these results were shown by Figure 9, which exist optimal lift coefficient with high lift to drag ratio in each h/c.

The lift to drag ratio increases at low AOA, then it decreases at high AOA that this increase mainly comes from an increase in the lift coefficient and reason of decrease is increase of the drag coefficient. The maximum lift to drag ratio for rectangular wing with Glenn Martin 21 section were determined at AOA=2°.

The overall computation time is very short by using the present work but the computation time of numerical simulation for rectangular wing in the fix AOA and h/c was about 10 min with an AMD Athelon (2.2 GHz) running Linux [21].

Also, the height static stability of WIG with AR=2 in the AOA=4° was examined and its result was depicted in Table 1.

According to Table 1, HS has positive sign then WIG is statically unstable. Because of the pitch aerodynamic center was located upstream of the height aerodynamic center. This result was estimable for static stability of WIG without horizontal tail. Also, Table 1 was shown that the value of $C_{l_a}$ and $C_{M_z}$ decreases with increase of h/c. the value of $C_{l_a}$ and $C_{M_z}$ decreases with increase of h/c.

Figure 10 was plotted variations of the height stability, the center of pitch and the center of height respect to h/c for rectangular wing with AR=2 according to results of Table 1.

According to Figure 10, the height aerodynamic center moves backwards and the pitch aerodynamic center moves forwards when h/c increases respect to leading edge. When the WIG moves out of ground effect, the aerodynamic centers (height and pitch) convert to unique center that this center is generally called aerodynamic center.

6. Conclusion

The practical mathematical modeling with low computational time and high accuracy was presented for calculation of the aerodynamic characteristics of rectangular WIG.

The RANS equations were converted to Bernoulli equation by reasonable assumptions. Also Helmbold's equation was developed for calculation of the wing's lift coefficient slope in ground effect by definition ARz. The present work could calculate aerodynamic...
characteristics of finite wing with symmetric and asymmetric airfoil section. The present work respect to the numerical methods and experimental methods has lower computational time and cost. The height static stability can be investigated easily by aerodynamic results of present work for WIG.

The results of present work have good agreement with the experimental results and maximum of errors for aerodynamic characteristics is below 13%. The present work results are better accurate than VLM, where the present work results are closer to the experimental result. The numerical results by CFD method is closer than the present work result to the experimental result but the present work has computation time very shorter than CFD method.

The aerodynamic characteristics of rectangular wing with Glenn Martin21 airfoil section at various ARs was calculated completely in ground effect. The lift coefficient of wing increases nonlinearly with reduction of h/c. Also the drag coefficient decreases with reduction of h/c due to reduction of induced drag coefficient. The aerodynamic performance of wing increases with reduction of h/c and also increases by increasing the AR. The pitching moment coefficient of wing increases with approximate to the ground.

It was confirmed that the wing with Glenn Martin21 airfoil section without horizontal tail cannot be satisfying the height static stability. It was understood that the height static stability increases with reduction h/c.

The practical method that presented in this study is applicable for optimization of aerodynamic performance of WIG craft and racing boat with aerodynamic surface in ground effect.

**Symbols**

- WIG = Wing-in-ground effect
- e = Chord of wing (m)
- h = Height of flight at trailing edge (m)
- b = Wing span (m)
- h/c = Ground clearance
- AOA = Angle of attack (deg)
- AR = Aspect ratio
- ARe = Equivalent aspect ratio
- P = Pressure (N/m²)
- F = Vertical force act on airfoil (N)
- Mairfoil = Moment of the airfoil (Nm)
- \( V_\infty \) = Free stream velocity (m/s)
- \( C_L \) = Lift coefficient
- \( C_D \) = Drag coefficient
- \( C_M \) = Pitching moment coefficient
- \( C_{Lc} \) = Lift coefficient at free stream
- \( C_L \) = Lift coefficient due to ground effect
- \( C_{Mc} \) = Moment coefficient at free stream
- \( C_M \) = Moment coefficient due to ground effect
- \( C_{Df} \) = Friction drag coefficient
- \( C_{Dp} \) = Pressure drag coefficient
- \( C_{Di} \) = Induced drag coefficient

- HS = Height Stability
- RANS = Reynolds averaged Navier-Stokes
- CFD = Computational fluid dynamics
- VLM = Vortex lattice method

**Greeks**

- \( \rho \) = Air density (kg/m³)
- \( \alpha \) = Angle of attack (deg)
- \( \sigma \) = Induced drag ground effect influence ratio
- \( \phi \) = Influence coefficient

7. Reference


19- Wieselsberger, C., (1922), *Wing resistance near the ground*, NACA, No. TM77.