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FEM Updating for Offshore Jacket Structures Using Measured Incomplete Modal Data

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ABSTRACT

Marine industry requires continued development of new technologies in order to produce oil. An essential requirement in design is to be able to compare experimental data from prototype structures with predicted information from a corresponding analytical finite element model. In this study, structural model updating may be defined as the fit of an existing analytical model in the light of measured vibration test. After fitting, the updated model is expected to represent the dynamic behavior of the jacket structure more precisely. In this way, current article presents a direct based updating study of a reduced scale four-story spatial frame jacket structure fabricated and tested at mechanical system and signal processing laboratory. Also, an efficient model updating process is presented with limited modal data, which uses modal data in order to improve the correlation between the experimental and analytical models.
The proposed technique is computationally efficient since it does not require iterations. It updates the mass and stiffness matrix such that they are compatible with the modal data of the observed modes.

1. Introduction

Jacket-type offshore platforms are by far the most common kind of marine structures and they play an important role in oil and gas industries in shallow and intermediate water depth such as Persian Gulf region. As offshore jacket structures require more critical and complex designs, the need for accurate considerations to determine uncertainty and variability in analytical models, loads, geometry, and material properties has increased significantly. In this context, one way to verify the math model accuracy is by comparing the experimental results provided through the conduction of dynamic tests with those expected from a previous analytical analysis [1-4]. Model updating is becoming a usual technique to improve the correlation between FEMs and measured data [5,6]. Based on the type of parameters that are updated and the measured data that is utilized, there are a number of procedures to the problem. The model updating problem has also applications in damage detection and integrity monitoring of the structures, such as jacket structure, bridges, highways, etc. [7-10]. The updating techniques can be generally categorized into three types: (a) direct matrix model updating techniques, (b) iterative techniques, and (c) frequency response techniques. The direct matrix updating approaches solve for the updated matrices by forming a constrained optimization problem. The excellence of direct techniques is that they are computationally straightway and efficient; since the result of the computation is unique, it is not necessary addressing the problem of whether the solution converges. For instance, Baruch assumed the mass matrix to be correct and updated the stiffness matrix [11,12].
An initially step estimated the mass normalized eigenvectors closest to the measured eigenvectors. Berman challenged whether the mass matrix should be considered exact, and updated both the mass matrix and the stiffness matrix [13,14]. Baruch explained these techniques as reference basis techniques, since one of three quantities (the measured modal data, the analytical mass and stiffness matrices) is assumed to be exact, or the reference and the other two are updated [15]. Caesar developed this approach and produced a range of techniques based on optimizing a number of cost functions [16]. All the techniques described so far, are similar in this aspect that only one quantity is updated at a time. Applying the measured modal data as a reference, Wei updated the mass and stiffness matrices simultaneously [17-19].
The constraints imposed were mass orthogonality, the equation of motion and the symmetry of the updated matrices. All of the aforementioned techniques used real mode shapes and natural frequencies. The

measured mode shapes were processed to create the equivalent real modes. This work focuses on improvement of stiffness and mass matrices by using Lagrange multiplier based techniques (direct technique) along with empirical study, so that the updated matrices reproduce the measured modal data. The experimental and the numerical modal analysis for obtaining the dynamic behavior of system have been implemented during this work. One main scope of the Experimental Modal Analysis (EMA) is extraction of the frequency response functions (FRF's). In the first step of an EMA, the elements of at least one full raw or one full column of the FRF matrix should be measured and then the natural frequencies can be identified using a variety of different methods such as Rational Fraction Polynomial method. In present study, this method has been applied using the ME'scope software. Another very important aspect of modal testing is the correlation and correction of a numerical model such as a FEMs. In the numerical modal analysis the governing equations are formed by obtaining mass and stiffness matrices and by solving them, the modal parameters can be estimated and finally the response of system is calculated. This means that the standard Eigen problem ($|[\mathbf{K}]-[\boldsymbol{M}]\boldsymbol{\omega}^{2}|=\mathbf{0}$) must be solved. Based on the basic concept of the vibration analysis, the natural frequency which resulted from the mentioned equation is an undamped frequency and in this problem the damping is not considered. Of course, the damping parameters play individual role in dynamic behavior of a real structure. But in most of the damage detection problems, in order that the methods would not be affected by damping, the undamped natural frequencies are considered as the desire extracted features via the FEM software. Of course, both the experimental and numerical methods can cause some errors in the measured frequencies, but in a FEM updating for the damage detection process, the experimental modal final results are far more acceptable and considered as the objective. Therefore the FEM is updated according to the experiment, so that the numerical natural frequencies approach to the experimental results. In addition, in this research, we propose an improved procedure for updating the stiffness and mass matrices. The direct matrix model updating technique works exclusively with only a small number mode shapes, and it is capable of preserving the mode shapes that are affected by updating. For example, if the model size is small, then all the mode shapes of the updated model can be computed explicitly using the existing computational techniques. However, this is not possible if the model size is large; because, the direct verification techniques of matrix computations are capable of computing only a few eigenvalues and eigenvectors.

The current paper (proposed methodology) for applying the incomplete experimental data uses only low-dimensional matrices, even though the FEM might be very large. In this regard, improved reduction algorithm is used. It is worth mentioning, in the structural health monitoring process using the characteristic parameters; the use of updated matrices based on experimental results is useful. On the other hand, in model updating of an offshore jacket platforms using EMA, there are two major challenges ahead: (a) the mismatching of measurement sensors and degrees of freedoms (DoFs) of the analytical model, namely the spatial incompleteness and (b) the unavoidable corrupted measurements [20, 21]. In dealing with spatially incomplete situations and the effects of noise to overcome uncertainty problem, we can use improved model reduction scheme and implement of precise experimentation. Furthermore, to overcome modeling uncertainty problem the FEM updating process is applied by using results of the experiment on physical model of the offshore jacket platforms, when limited, spatially incomplete modal data is available. In this study, the presented reduction technique removes the bad effect of model reduction process on the model updating procedure by adding a correction term (inertial effects) to the formula of the simplest reduction schemes. The FEM updated provides a useful and less expensive way for studying the fixed marine structures. Thus, experimental programs are necessary to provide validation for the results and reduce the uncertainty of the values of the excitations for of fixed marine structures.

2. The Model Updating Methodology

2.1 Lagrange Multiplier: Updating of Stiffness Matrices

The Lagrange multiplier based techniques (direct technique) generally consider one parameter set, either mass or stiffness to be correct, and the remaining two that is either mass or stiffness, and the modes, are updated by minimizing a cost function with the appropriate constraints imposed through Lagrange multipliers. Modes that are measured from the structure will not necessarily be orthogonal to the analytical mass matrix since there are likely fewer transducers than DOF and because of imperfect measurements. For the direct techniques that assume that the mass matrix is correct, it is usually difficult to enforce orthogonality.

In order to ensure the eigenvectors are orthogonal, the measured eigenvectors must be corrected. Baruch has derived a cost function, J, (1), in which the newly updated eigenvector matrix φ is to be minimized [11]:

$$J=\left\|N(\varphi-\varphi_m)\right\|=\sum_{i=1}^{n}\sum_{k=1}^{m}\left[\sum_{j=1}^{n}[N]_{ij}\left([\varphi]_{jk}-[\varphi_m]_{jk}\right)\right]^2 \quad (1)$$

Where $N = M_a^{1/2}$, M_a is the analytical mass matrix, φ_m is the measured eigenvector , $[N]_{ij}$, $[\varphi]_{ij}$, $[\varphi_m]_{ij}$ are the *(i,j)* elements of the matrices N, φ, φ_m, m is the number of measured eigenvectors, n is the number of DOF in the analytical model. Subjected to the orthogonality condition:

$$\varphi^T M_a \varphi = I \tag{2}$$

The Lagrange Multiplier technique uses the constraint (2) to produce the augmented function to be minimized as [6]:

$$J = \sum_{i,h,j=1}^{n} \sum_{k=1}^{m} \left\{ [N]_{ij} \left([\varphi]_{jk} - [\varphi_m]_{jk} \right) [N]_{ih} \left([\varphi]_{hk} - [\varphi_m]_{hk} \right) \right\} + \sum_{i,h=1}^{m} \gamma_{jh} \left\{ \sum_{j,k=1}^{n} \left([\varphi]_{ji} [M_a] [\varphi]_{kh} - \delta_{ih} \right) \right\} \tag{3}$$

Where the terms, $\boldsymbol{\gamma_{jh}}$, are the Lagrange multipliers, which are cast into a matrix $\boldsymbol{\Gamma}$, and the terms $\boldsymbol{\delta_{ih}}$ represent errors. The Lagrange Multipliers may be forced to be unique by introducing the constraint of symmetry so that:

$$\Gamma = \Gamma^T \tag{4}$$

Differentiating the augmented function (3) with respect to each element of the corrected eigenvector matrix and the following expression is found:

$$\varphi = \varphi_m [I + \Gamma]^{-1} \tag{5}$$

When substituted back into the orthogonality condition, becomes:

$$[I + \Gamma]^{-1} \varphi_m^T M_a \varphi_m [I + \Gamma]^{-1} = I \tag{6}$$

By pre and post multiplying by $[I + \Gamma]$and taking the square root, it becomes:

$$[I + \Gamma] = \left[\varphi_m^T M_a \varphi_m \right]^{0.5} \tag{7}$$

Finally, substituting Equation (7) into (5), the Equation for the corrected eigenvector matrix is:

$$\varphi_u = \varphi_m \left[\varphi_m^T M_a \varphi_m \right]^{-0.5} \tag{8}$$

If it is assumed that the analytical mass matrix is already correct and the eigenvectors are corrected to ensure orthogonality, the stiffness matrix can now be updated. Baruch found that the updated stiffness matrix can be found to minimize the cost function [11]:

$$J = \frac{1}{2} \left\| N^{-1} (K - K_a) N^{-1} \right\| \tag{9}$$

$$J = \frac{1}{2} \sum_{ij=1}^{n} \left[\sum_{h,k=1}^{n} \left[N^{-1} \right]_{jh} \left([K]_{hk} - [K_a]_{hk} \right) \left[N^{-1} \right]_{kj} \right]^2 \tag{10}$$

Where $N = M_a^{1/2}$, $[N^{-1}]_{ij}$, $[K]_{ij}$, $[K_a]_{ij}$ are the (i,j) elements of the matrices N^{-1}, K, K_a, and is subject to the two constraints:

$$K\varphi = M_a \varphi \Lambda$$

And

$$K^T = K \tag{11}$$

Λ represent the eigenvalue matrix. The cost function is then differentiated with respect to the updated stiffness matrix and results in the following Equation:

$$M_a^{-1} (K - K_a) M_a^{-1} + 2\Gamma_\Lambda \varphi^T + 2\Gamma_k = 0 \tag{12}$$

Where Γ_Λ and Γ_k are Lagrange Multipliers. By calculating the values of the Lagrange Multipliers, substituting them into Equation (9), and then rearranging Equation, the updated stiffness matrix can be found using the following Equation:

$$K_u = K_a - K_a \varphi_u \varphi_u^T M_a - M_a \varphi_u \varphi_u^T K_a + M_a \varphi_u \varphi_u^T K_a \varphi_u \varphi_u^T M_a + M_a \varphi_u \Lambda \varphi_u^T M_a \tag{13}$$

2.2 Lagrange Multiplier: Updating of Stiffness and Mass Matrices

Berman and Nagy used a similar approach to the one presented by Baruch, however, they used it to update both the mass and stiffness matrices by assuming that the measured eigenvector matrix is correct [14]. The advantage of this scheme is that it is not necessary to calculate the corrected eigenvectors because the mass matrix is updated in such a manner to ensure the orthogonality of the eigenvectors to the mass matrix. Given the analytical mass matrix, M_a, and the measured eigenvector matrix, φ_m, the following cost function is created in which the updated mass matrix is found to minimize the function:

$$J = \frac{1}{2} \left\| M_a^{-\frac{1}{2}} (M - M_a) M_a^{-\frac{1}{2}} \right\| \tag{14}$$

This function is also subject to the orthogonality constraint:

$$\varphi_m^T M \varphi_m = I \tag{15}$$

The cost function J is minimized using the same steps as the cost function containing the corrected stiffness matrix. The result is:

$$M_a^{-1}\left(M - M_a\right)M_a^{-1} + \varphi_m \Gamma\ \varphi_m^T = 0 \qquad (16)$$

Combining this Equation with that of the orthogonality constraint and the Lagrange Multiplier, the updated mass matrix can be found by adding an updating term, the second term in Equation (16), to the analytical mass matrix as follows:

$$M_u = M_a + M_a \varphi_m \overline{M}_a^{-1}\left(I - \overline{M}_a\right)\overline{M}_a^{-1} \varphi_m^T M_a \qquad (17)$$

Where, $\boldsymbol{\overline{M}_a^{-1} = \varphi_m^T M_a \varphi_m}$.

The updated mass matrix can now be used to calculate the updated stiffness matrix. Since the eigenvector matrix is orthogonal to the newly updated mass matrix, the calculation for the updated stiffness matrix from the previous section can be used; however, the newly acquired updated mass matrix and the measured eigenvector matrix will appear in place of the analytical mass matrix and the corrected eigenvector matrix. So the Equation for the updated stiffness matrix becomes:

$$\begin{aligned} K_u = K_a - K_a \varphi_m \varphi_m^T M_u - M_u \varphi_m \varphi_m^T K_a + \\ M_u \varphi_m \varphi_m^T K_a \varphi_m \varphi_m^T M_u + M_u \varphi_m \Lambda\ \varphi_m^T M_u \end{aligned} \qquad (18)$$

3. Improved Reduction Algorithm Due to Incomplete Modal Data

One of the simplest reduction schemes is static reduction (Guyan). The full scale model may have certain nodal freedoms specified as master freedoms. The remaining freedoms are slave freedoms. For dynamic analysis purposes the mass, stiffness and loading on the slave freedoms are condensed to these master freedoms. In matrix notation the overall matrices may be partitioned into master, slave and cross coupling terms.

$$\begin{aligned} \begin{bmatrix} [M_{mm}] & [M_{ms}] \\ [M_{sm}] & [M_{ss}] \end{bmatrix} \begin{Bmatrix} \ddot{X}_m \\ \ddot{X}_s \end{Bmatrix} + \\ \begin{bmatrix} [K_{mm}] & [K_{ms}] \\ [K_{sm}] & [K_{ss}] \end{bmatrix} \begin{Bmatrix} X_m \\ X_s \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \end{aligned} \qquad (19)$$

Where, the subscripts m and s refer to the master and slave coordinates, respectively. The technique then ignores the inertia terms in the second set of Equations. Neglecting the inertia terms for the second set of equations we have:

$$[K_{sm}]\{X_m\} + [K_{ss}]\{X_s\} = [T_s]\{X_m\} \qquad (20)$$

By eliminating the slave DOF, we obtain:

$$\begin{Bmatrix} X_m \\ X_s \end{Bmatrix} = \begin{bmatrix} [I] \\ -[K_{ss}]^{-1}[K_{sm}] \end{bmatrix} \{X_m\} = [T_s]\{X_m\} \qquad (21)$$

T_s is Guyan transformation matrix and I is identify matrix.

The reduced Guyan mass and stiffness matrices are then given by [22]:

$$[M_R] = [T_s^T][M][T_s] \qquad (22)$$

$$[K_R] = [T_s^T][K][T_s] \qquad (23)$$

For larger marine structures, where it is necessary to reduce many slave DOF, this technique will not be as accurate as some of the more advanced approaches. Accordingly, improved reduction skill is probably the best practical process for solving large dynamic problems. Only the smallest frequencies are usually excited and for a typical jacket no more than 30 would normally be required.

The process known as the Improved Reduction System (IRS) was presented by O‟Callahan in 1989 [5]. This technique is an improvement over the Guyan static reduction scheme via introducing a term that includes the inertial effects as pseudo static forces. A transformation matrix T_i is applied to reduce the mass and stiffness matrices. It is defined as:

$$[T_i] = [T_s] + [S][M][T_s][M_R^{-1}][K_R] \qquad (24)$$

where

$$S = \begin{bmatrix} [0] & [0] \\ [0] & [K_{SS}^{-1}] \end{bmatrix} \qquad (25)$$

M_R and K_R are the statically reduced mass and stiffness matrices.

The new reduced mass and stiffness matrices can be obtained by:

$$[M_{IRS}] = [T_i^T][M][T_i] \qquad (26)$$

$$[K_{IRS}] = [T_i^T][K][T_i] \qquad (27)$$

For this process, the rows and columns corresponding to the slave coordinates are eliminated from the mass and stiffness matrices one at a time; this allows the mass and stiffness matrices to adapt to the removal of a slave, and can possibly alter the DOF that will be removed. After each reduction, the DOF with the lowest $\boldsymbol{K_{ii}}/\boldsymbol{M_{ii}}$ term is the slave which will be eliminated next [23].

4. Feature Extraction of Structure

4.1 Theoretical Modal Analysis

Modal analysis is the procedure of identifying the intrinsic dynamic properties of a system in forms of natural frequencies, damping factors and mode shapes, and using them to formulate an analytical model for its dynamic behavior. In this paper, the ***Block Lanczos*** method has been applied for solving the modal analysis.

4.2 Modal Testing

Modal testing is an experimental method utilized to derive the modal model of a linear time-invariant vibration system. The modal testing methods have been widely applied to assess the dynamic characteristics of the structures. The method's common applications include not only feature extraction but also structural integrity monitoring of structures. Modal testing processes are offered by three main procedures: Experimental Modal Analysis (EMA) and Operational Modal Analysis (OMA). In the EMA method, structures are excited by certain forces and the responses of the structures are recorded. The structural modal parameters are extracted from the identified modal models based on the recorded input/output. The OMA method requires only the outputs to be measured for the construction of a modal model for structures, so artificial excitations are not necessary.

The theoretical basis of the method is secured upon establishing the relationship between the vibration response at one location and excitation at the same or another location as a function of excitation frequency.

In the present article, ***Modal Assurance Criterion*** **(MAC)** method is applied for updating of the platform model. The modal assurance criterion, which is also known as mode shape correlation coefficient, between analytical mode φ_i and experimental mode φ_j is defined as:

$$MAC(\varphi_i, \varphi_j) = \frac{\left|\varphi_i^T \varphi_j\right|^2}{\left(\varphi_i^T \varphi_i\right)\left(\varphi_j^T \varphi_j\right)} \tag{28}$$

A *MAC* value close to 1 suggests that the two modes are well correlated and a value close to 0 indicates uncorrelated modes [24, 25].

5. Description of The Physical Model and Test Set Up

5.1 Necessary Conditions to Select the Physical Model

Most offshore platform topsides and jackets are transported to the final site by barge. Offshore platforms are commonly moved long distances to install in the site. The transportation phase can be critical for some permanent members and determines the design of the sea fastenings and other temporary attachments. Large forces can be generated, particularly by the roll and pitch rotations and accelerations and the heave acceleration of the barge. This is especially the case for jacket platforms where the length of the legs results in large bending moments from roll and pitch motions. These effects can cause significant fatigue damage as well as possibly overstressing the legs. Parts of the structure which overhang the barge deck may experience buoyancy, drag and inertia loading and also, possibly more seriously, slam loading if the member penetrates the water surface. The sea area, barge size, season and duration of the tow should be taken into account when selecting the environmental conditions to be used for the analysis. As a result, the step of loading is very important and it has a significant effect on the design of jacket platforms. Thus, the physical model is constructed according to this loading condition. Accordingly, the model tests were performed in dry.

5.2 Physical Model and Laboratory Work

The case study is a fixed jacket-type offshore platform model and the geometric dimensions of the structural members are similar to those in the model first used by Huajun et al. [26]. The general shape of the model represents a space frame with four main legs that are connected to the top deck. The frame of the model has horizontal and diagonal bracing members at all stories. The space steel frame jacket structure, consisting of 46 steel tubular members with outer diameter 18 mm, wall thickness 2.5 mm for leg members and outer diameter 12mm, wall thickness 1.5 mm for other members, is fixed at the ground.

A physical model was constructed of stainless steel pipes that were welded together using argon arc welding to ensure proper load transfer. The mass density of the members is 7850 kg/m3 and the Young’s modulus of steel is 207 GPa. There are 16 nodal points in the FEM, three translational DoFs at each node, thus total 48 translational DoFs. The physical model and FEM of the laboratory tested space steel frame jacket structure adopted for model updating is shown in Figure 1. The white noise signals were used as the input exciting signal. The instrumentation included two light uniaxial accelerometers (4508 B&K) in both the X and Y directions on each joint for response measurement and a load cell for measuring the excitation force.

The frequency sampling of the test setup was chosen to be 10 kHz, and the frequency range was 0-200 Hz. The recorded data were sent to the PULSE [27] software package for processing. The data required for calculating the FRFs were recorded by sensors that were fixed on the physical model joints. Because there were more desired points for measurement (i.e., the 16 joints of the model) than the number of available channels and accelerometers, the measurements were performed in 16 steps. The test rig and instruments are illustrated in Figure 2.

6. Results and Discussion

6.1 Improvement of Stiffness Matrix

The jacket platform was modeled using 3D FE software, *ANSYS* and modal analysis was performed.

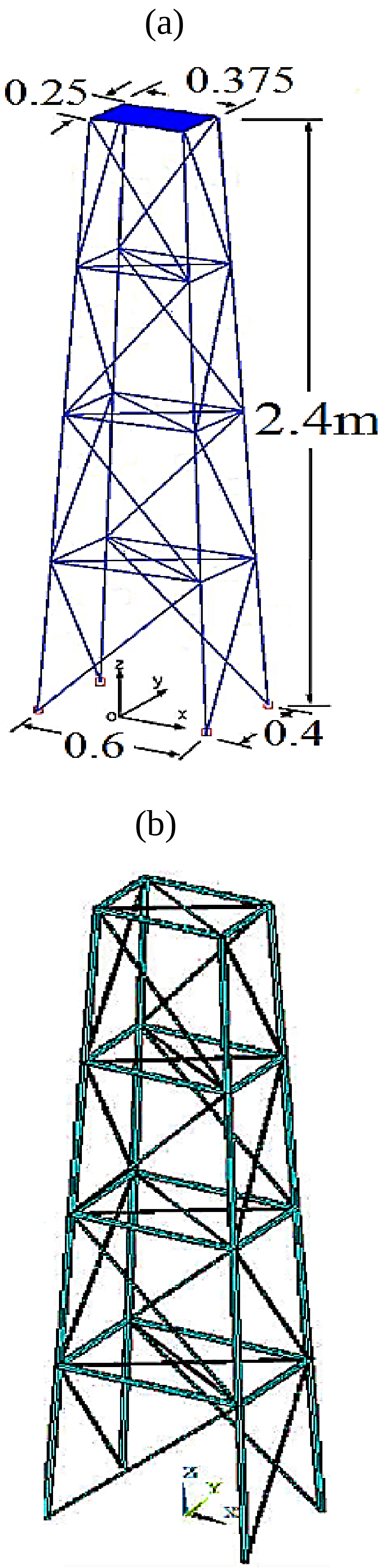

Figure 1. (a) The physical model and (b) finite element model of the jacket

For the implementation of the proposed technique, initially the mass and stiffness matrices were extracted by *ANSYS* software under ***SUBSTRUCTURE*** analysis (see Figure 3).

With limited transducers, it is only possible to estimate the lower modes. Mode shapes of the numerical and EMA are shown in Figure 4; also frequencies of numerical, experimental and updated model along with *MAC* value are listed in Table 1. In this case the corrected stiffness matrix becomes similar to Figure 5. It is apparent that the updated stiffness matrix is now filled and no longer physically represents the model. Finally, it can be concluded that there is perfect correlation between the numerical and experimental modal vectors. This means that, *MAC* value is close to 1 and the numerical and experimental models have appropriate correspondence.

6.2 Improvement of Stiffness and Mass Matrices

Using Equations (17) and (18), the updated mass and stiffness matrices will be similar to Figure 6. Again, the updated matrices become completely filled for the second case. However, since both the mass and stiffness matrices are allowed to be perturbed, they are closer to physically representing the system. The results for the first 4 modes are presented in Table 2. Also, the Changes of those matrices are shown in Figure 6. The results are similar to the initial direct technique (stiffness), which is to be expected, since they are both based on similar optimization procedures, the only difference is in the matrices being updated. However, it is noted that the mass matrix is no longer diagonal; since the stiffness matrix is already not a physical representation it is more beneficial to update only the stiffness matrix.

7. Conclusions

FE matrix updating has attracted a notable amount of attention by the engineering community and consequently, there now exist a voluminous work on this problem. In this research the ability of empirical investigation of the jacket platform model updating are considered. Also, an efficient model updating technique was presented with incomplete modal data, which uses modal data in order to improve the correlation between the experimental and analytical models. An example with incomplete modal data of a typical reduced scale four-story spatial frame of the jacket platform was carried out showing that the methodology was able to correct update both mass and stiffness matrices and reproduce correctly the tested data. The mode shapes are not required to be measured at all DOF. The proposed technique removes the bad effect of model reduction process on the model updating procedure.

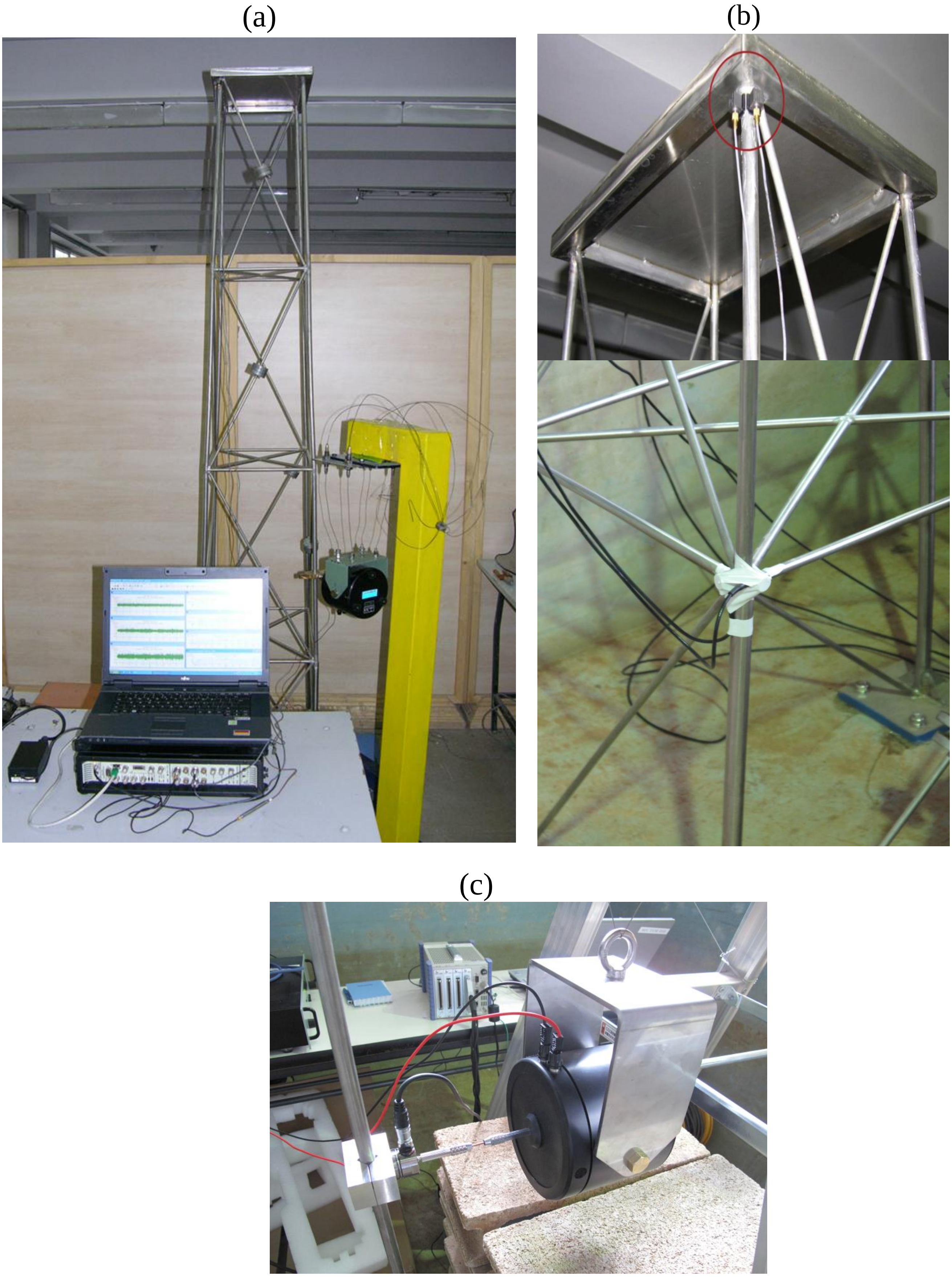

Figure 2. (a): modal testing and tested three-dimensional structure, (b): Installed accelerometers and sensor position and (c): The experimental test rig and instruments.

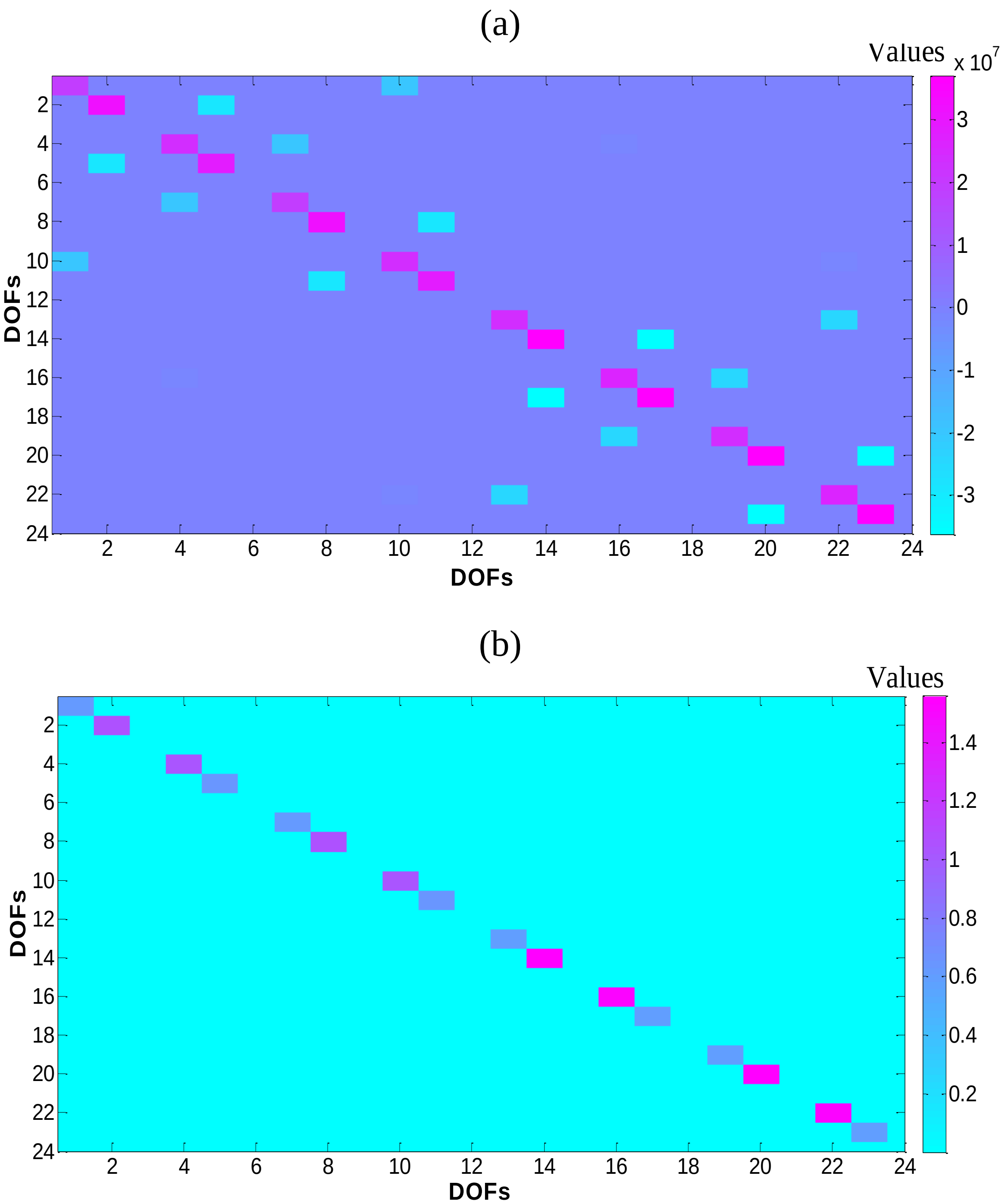

Figure 3. (a) Initial stiffness matrix (b) Initial mass matrix of platform model

Table 1: The first four natural frequencies (Updated Stiffness Matrix)

Mode no.	Natural frequencies (Hz)			Differences (%)	MAC
	Numerical Analysis	Experimental Result	Updated model		
1	67.29	58.34	58.83	0.85	0.994
2	91.46	94.13	93.61	0.50	0.992
3	100.8	106.21	106.85	0.60	0.990
4	125.1	130.28	131.04	0.58	0.992

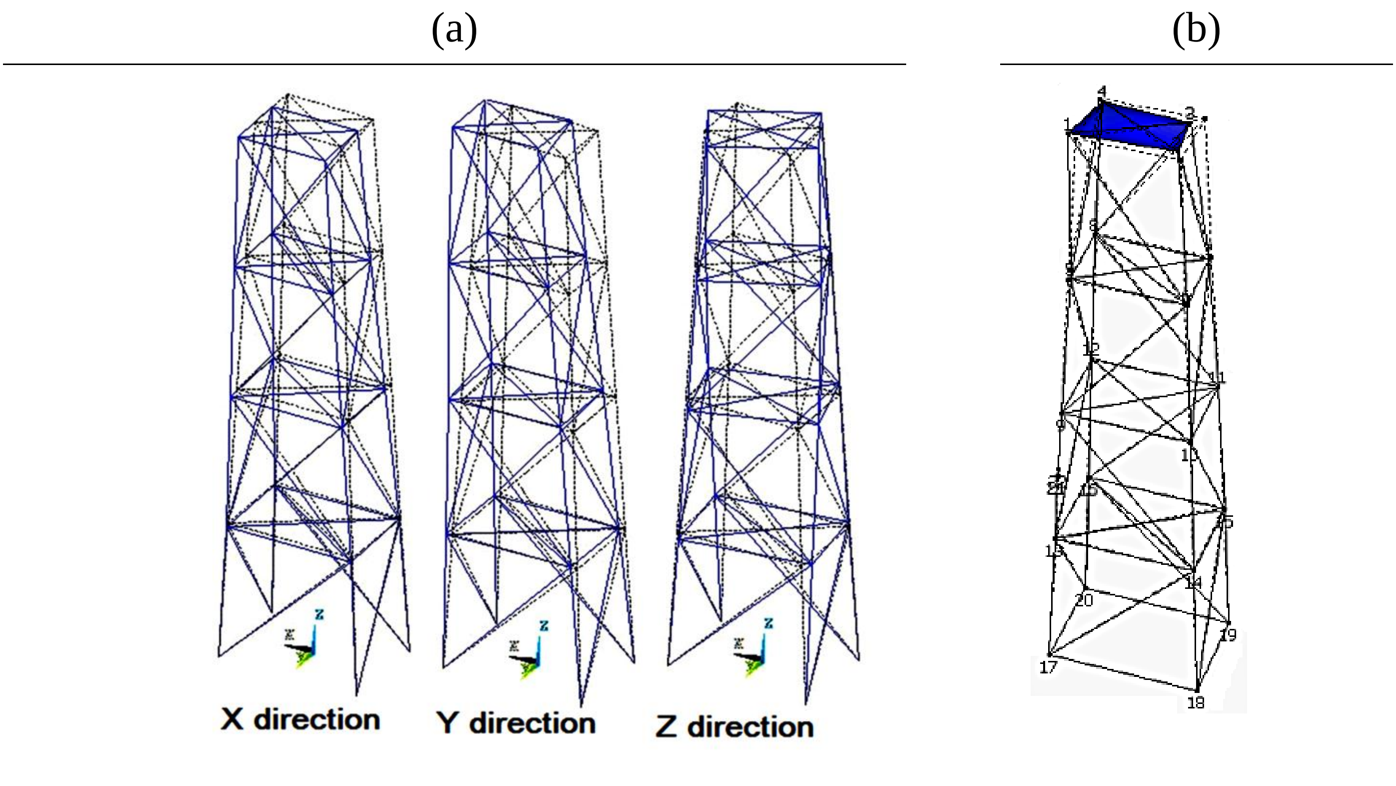

Figure 4. The first mode shape using (a) numerical modal analysis, (b) experimental modal analysis.

Table 2: The first four natural frequencies (Updated Stiffness and Mass Matrices)

Mode no.	Natural frequencies (Hz)			Differences (%)	MAC
	Numerical analysis	Experimental result	Updated model		
1	67.29	58.34	58.59	0.4	0.995
2	91.46	94.13	93.84	0.3	0.992
3	100.8	106.21	106.6	0.4	0.994
4	125.1	130.28	129.87	0.4	0.991

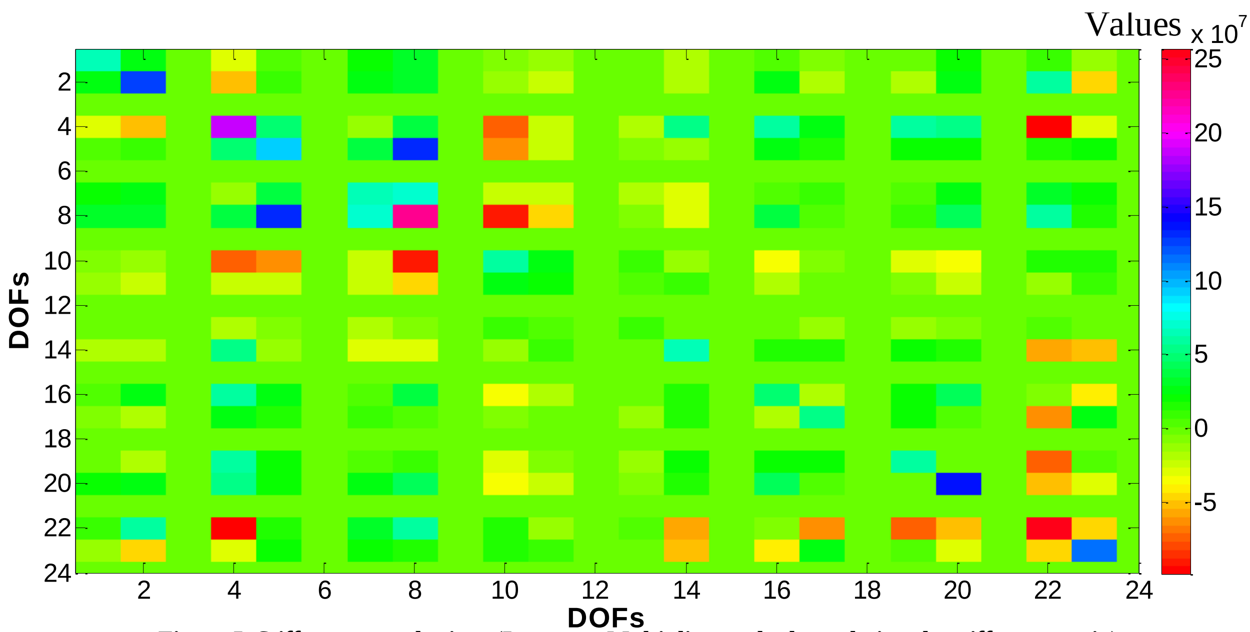

Figure 5. Stiffness perturbation: (Lagrange Multiplier method - updating the stiffness matrix).

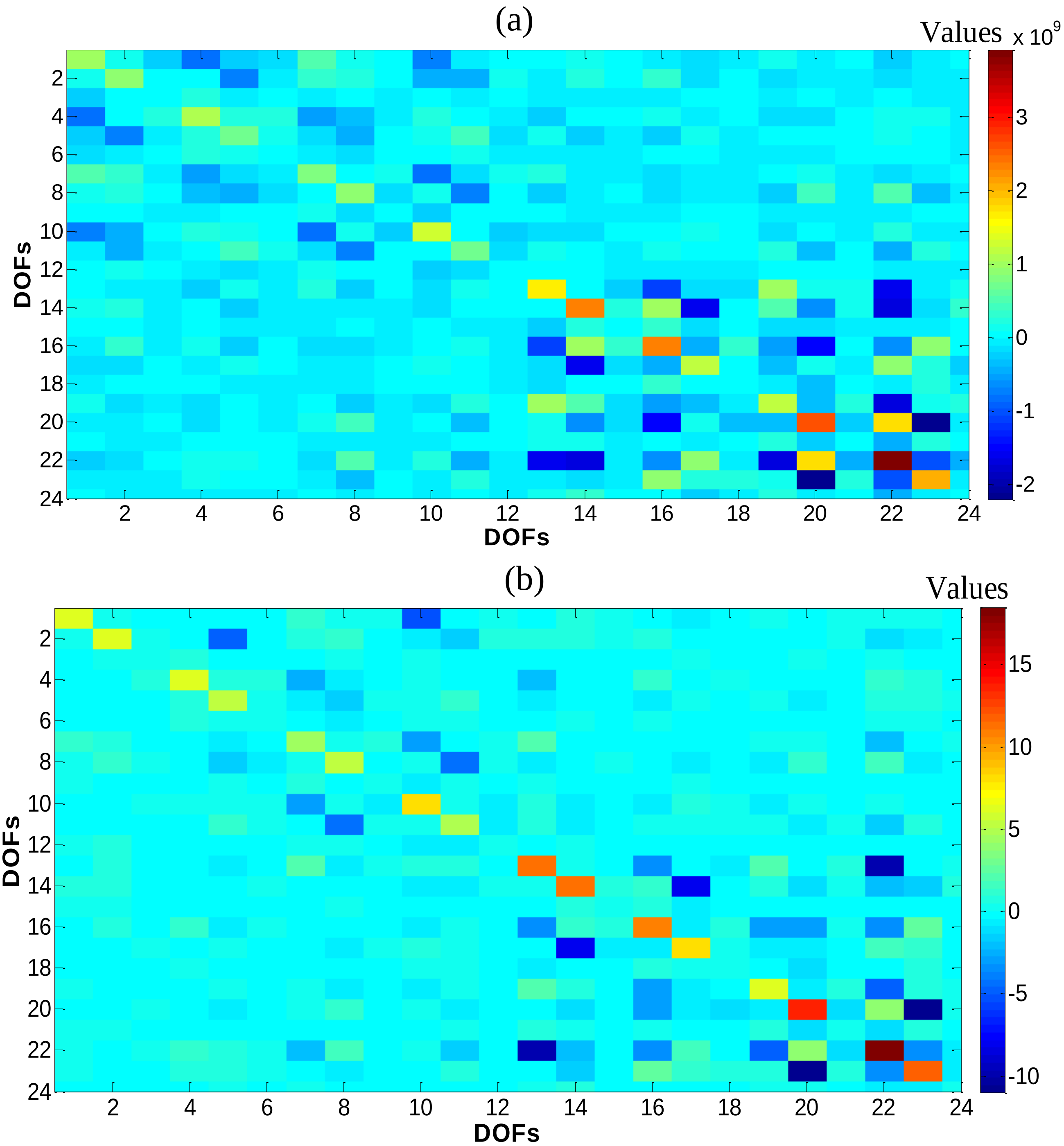

Figure 6. (a) Stiffness perturbation and (b) Mass perturbation: (Lagrange Multiplier method - updating the stiffness and mass Matrices).

The proposed technique is computationally efficient since it does not require iterations. It updates the mass and stiffness matrix such that they are compatible with the modal data of the observed modes. The Lagrange multiplier techniques reproduce the measured eigen-system, however, the results are not physically meaningful, or in other words cause the updated system to lose its physical representation. This is a potential problem for situations where the stiffness and/or mass of a specific DOF are needed, such as in damage detection. In the structural health monitoring process using the characteristic parameters, In other words, the use of updated matrices based on experimental results is useful. These techniques are advantageous for systems that contain measured eigenvalue and eigenvectors for every DOF, especially if the physical representation of the mass and stiffness matrices is not of importance. The FEM updating provides a practical and less expensive way for studying the behavior of fixed offshore platforms. However, an experimental program can be used to validate a FEM. Through experimentations one can reduce the uncertainty of the fixed offshore platforms.

List of Symbols

EMA Experimental modal analysis
IRS Improved reduction system
M_a Analytical mass matrix
M_R Statically reduced mass matrix
K_a Analytical stiffness matrix

K_R	Statically reduced stiffness matrix
K_u	Updated stiffness matrix
φ_m	Measured eigenvector
φ_u	Corrected eigenvector matrix
Γ_k, Γ_Λ	Lagrange Multipliers
δ_{ih}	Errors
γ_{jh}	Lagrange multipliers

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