Assessment of Semi-Active Tunes Mass Damper Application in Suppressing Seismic-Induced Vibration of an Existing Jacket Platform

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1. Introduction
Increasing need of the fossil fuel according to onshore energy sources limitations, coerced into invest on excavation of hydro carbonic fuels derived from offshore sources day after day. It caused significant development in offshore industries since its inception about six decades ago. Being suitable to be constructed in water depth from a few meters to more than 300 m according to its characteristics, the steel jacket platform-a typical type of fixed offshore platform-is the most common in use among the various types of offshore structures. It is usually built from tubular steel members. The major structural components of such an offshore platform are jacket, piles and deck. A jacket structure which serves as bracing for the piles against lateral loads is fixed by piles driven through the inside of the legs of the jacket structure and into soil for many tens of meters. The deck structure is fixed upon the jacket structure [1-3]. Seismic excitations on one hand and harsh surrounding environmental conditions of oceans on the other hand, affect not only the routine operation of an offshore platforms (i.e. drilling and production), but also the safety and serviceability of the structure which lead to working crew discomfort and equipment malfunction. Reduced capacity generated by dynamic vibration caused by dynamic loads including earthquake as well as wind, wave and current, if not be taken into consideration appropriately finally leads to failure. Furthermore, the original design life of these platforms is about 25 years which many of them are beyond this life. As one of the main barriers in developing the offshore industry is vast expenses of platform constructions and its equipment; it is important to employ mechanisms to mitigate the structural responses and increasing the life time of the structures. Finally such mechanisms lead to optimal and economical design. Structural vibration control approach in offshore platforms is a big challenge to be encountered. Vibration control technologies, initially proposed to protect tall buildings and long span bridges against seismic and wind excitation, have achieved significant success in vibration mitigation of land-based structures. Nevertheless, they have rarely been applied to offshore platforms. Structural control mechanisms are divided up into passive, active, hybrid and semi active. Passive control devices do not require external energy but they have an inherent limitation. On the other hand, an active control mechanism, employing a set of actuators and sensors connected by a feedback loop, can be effective over a wide frequency range with the desired reduction in the dynamic response but it requires large external power...
source. The main value of feedback is to decrease the output sensitivity to parameter variations and also to attenuate the disturbance effects within the bandwidth of the control system. Semi active control driven by the reduced cost as compared to active control has been a growing interest, due to the absence of a large power actuator. A semi active device can be defined as a passive device in which the properties such as damping, stiffness,…can be varied in real time with a low power input. Semi active mechanisms are inherently passive and unlike active devices cannot threaten the system; they are also less vulnerable to power failure. These devices have the capability to vary the resistance law in some way so as to achieve a strong dependence of the control law on the relative velocity. The variable resistance law can be achieved in a wide variety of forms, as for example position controlled valves, rheological fluids (both magneto-rheological and electro-rheological), or piezoelectrically actuated friction [4-8].

Having the advantages of both passive and active control (i.e. reliability and smaller external energy requirements), semi active control mechanisms are being considered in offshore structures recently.

Relevant vibration control technologies have been applied to offshore structures in the following cases.

Using storage tanks as tuned liquid damper (TLD) on a fixed offshore platform to suppress the vibration of the structure subjected to random wave forces have been proposed by Vandiver and Mitome [9]. Kawano and Venkataramana [10] and Kawano [11] investigated the application of an active tuned mass damper (ATMD) to reduce wave-induced vibration of the platform and found it quite effective. The application of curtain active and passive control mechanisms in order to mitigate the dynamic response of steel jacket platforms due to wave-induced loading has been analyzed by Abdel-Rohman [12]. Employing stochastic analysis for offshore platforms, Lee [13] demonstrated the efficiency of mechanical added dampers so as to mitigate the in-line random wave forces’ vibrations. The effectiveness of an active control system for articulated leg platforms in view of diminishing the wave-induced dynamic responses has been taken into consideration by Sunjea and Datta [14,15]. Gattulli and Ghanem [16] developed an adaptive control technique employing an active TMD for suppression of vortex-induced vibrations of offshore structures. Terro et al. [17] applied a multi-loop feedback-control to an offshore jacket platform subjected to wave induced forces. Result demonstrates the effectiveness of this control method.

Focusing on suppressing the ice-induced vibration of Bohai Sea offshore platforms, Ou et al. [18] numerically investigated the ice-induced structural vibrations using in-situ measured ice force data, and also conducted numerical and experimental studies on ice-induced vibration control of offshore structures by adding viscoelastic dampers. Suhardjo and Kareem [19] analyzed the hybrid control in frequency domain utilizing active mass damper as well as passive tuned mass damper. Vibration control of offshore platforms due to wave-induced excitations utilizing magnetorheological (MR) dampers has been studied by Wang [20]. Smart isolation systems of offshore platform jacket structure using MR dampers have been proposed by Ou and Yang [21]. The isolators and MR dampers considered to be installed on the layer between the jacket structure and the upper deck of JZ20-2MUQ platform in Bohai Sea. Mahadik and Jangid [22] analyzed the response of offshore jacket platforms with an active TMD under wave-induced loading. Employing H2 Algorithm, Li et al. [23] designed an active TMD and evaluated its application under wave random loads.

An AMD with H2 control algorithm has been used in order to control the vibration of offshore platforms under wave-induced excitations by Ji and Meng [24]. Numerical results demonstrate the feasibility and effectiveness of the proposed method. Golafshani et al. [25] studied the effect of employing energy damper systems-especially friction dampers-in order to retrofit of offshore platforms. Besides, they analyzed the performance of jackets equipped by FDD and TMD separately so as to seismic vibration control plus fatigue damage control, the results compared with hybrid system and recognized efficient [26].

Yue et al. [27] used TMD in order to mitigate ice-induced jacket platform vibration in Bohai Sea and find this supplemental device favorably effective. The control forces derived from a LQG algorithm were employed to generate the training patterns for the modified probabilistic neural network (MPNN) in order to control the responses of offshore structures under random ocean waves by chang et al. [28].

Tabeshpour et al. [29,30] investigated the performance of TMD so as to suppress the dynamic responses of jacket and tension leg platforms. The effectiveness of the damping system employing a friction damper device (FDD) in a jacket platform is evaluated numerically by Komachi et al. [31]. FDD is used for seismic retrofit of a steel jacket platform located in Iranian waters of the Persian Gulf. Results indicated that responses of jacket reduce dramatically. Sudip and Datta [32] employed semi active control in a fixed offshore platform using LQR algorithm. The control devices are the semi active hydraulic dampers (SHDs), installed in the bracings of the jacket structure. The result signified to the fact that significant reductions in the peak values of response quantities of interest can be achieved by semi-active control, using SHDs. Employing H2/LQG algorithm, Taghikhany et al. [33] studied the effect of magnetorheological (MR) fluid dampers in seismic-induced dynamic response control of Sirri jacket
located in Persian Gulf. The results showed that the earthquake-induced vibrations can be effectively suppressed.

Analytical studies were undertaken to investigate the idea of using HDS variants in offshore jacket platforms with float-over deck in order to control both seismic vibration and fatigue damage by Jafarabad et al. [34]. Jakšic et al. [35] outlined the use of a tuned single liquid column damper for the control of a tension leg platform supported wind turbine. The reduction in peak and RMS values for displacement, velocity, and acceleration of the platform has been reported in this research. Based on a 1:200-scale model of an actual four-column jack-up offshore platform, Wu et al. [36] concentrated on the high response performance of a simple passive TMD system, and numerical and experimental investigations were performed. The results indicated that a reduction of >27% for the maximum peak displacement response and a reduction of >32% for the acceleration response were achieved.

As it is seen, although there had been several studies for passive and active control mechanisms effectiveness in jacket platform response suppressions, however few of them reported semi active control mechanisms. Besides, although Persian Gulf has not been considered as an active seismicity zone, but recent earthquakes in south of Iran at close distance of Sirri oil field located in Persian Gulf revealed that offshore jacket platforms in this area might suffer from severe vibrations during earthquake as well as wave-induced vibrations. Despite the presence of more than 135 offshore platforms in Persian Gulf and resistance necessity of them, almost none of the relevant studies considered the importance of seismic excitation as much as wave nevertheless.

2. Overview of Nosrat Jacket Platform
Belonged to Sirri oilfield, Nosrat jacket platform is located in the Iranian territorial water of Persian Gulf, approximately 100 km from the coast and approximately 32 km south-west of Sirri Island. The studied Nosrat jacket platform, illustrated in figure 1, is a wellhead platform which has fourteen slots. This part of Sirri oilfield was developed between 1975 and 1978 and is on stream since June 1978. Nosrat jacket platform is a steel truss deck, supported by a jacket structure with battered legs. The jacket is a six-legged steel structure with through-the-leg steel piles. The legs are battered and are supported laterally with a V-brace pattern. The bed line elevation is 60 m below sea level and the main/upper deck is located in 16.7 m above water surface. Figure 1 shows Nosrat Jacket.

3. Modeling and Analysis of Nosrat Jacket
The assessment analyses of Nosrat Jacket are linear and have been performed using SACS suite of program. These assessment analyses are consist of In-Place analysis, Pile-Soil Interaction analysis and Dynamic analysis. A numerical model of Nosrat offshore jacket platform with 198 DOFs using SACS (Structural Analysis Computer System) software was set up. The Morison equation was employed in order to estimate wave load [1], [37,38]. According to the fact that structure's dynamic behavior is significantly affected by the soil characteristics, which is a very weak layer of clay in Persian Gulf region. Numerical modeling was done using a lumped mass technique. Considering the effective length, effective cross-sectional area and effective moment of inertia of piles due to non-static soil-pile interaction, the dynamic characteristics of Nosrat Jacket platform were analyzed employing SACS software. As a consequence, mass and stiffness matrices as well as structural frequencies and mode shapes were derived. The modeled jacket in SACS software is presented in figure 2. Nosrat jacket frequencies belonged to dominant modes of vibrations accompanying with mass participation factors is showed in Table 1.
4. SATMD Design and location

The concept of the Tuned Mass Damper dates back to the 1940’s [47]. It consists of a secondary mass with properly tuned spring and damping elements, providing a frequency-dependent hysteresis that increases damping in the primary structure. TMD is a feasible and practical approach to reduce seismic and environment vibrations. Due to variation of dynamic properties of structure during earthquake, the efficiency of TMD system which has been optimally designed could be diminished. Since the inherent limitations of passive devices have been proved for example passive TMD can be tuned for first modal frequency not wider frequency range- one of the strategies which can be helpful to preserve TMD advantageous is semi active control. Semi-active tuned mass damper (SATMD) combines a passive TMD and a semi-active damper (here a MR damper) to generate the desired control force and adjusted TMD online with structure dynamic behavior [39-42]. Employed in MR devices, magneto-rheological (MR) fluids, with a considerable yield stress, exhibit switching as fast as the order of millisecond. Such characteristics makes MR fluids excellent contenders for semi-active devices [4].

As mentioned previously, SATMD system utilized here, is composed of a passive tuned mass damper (TMD) and two magneto-rheological (MR) dampers in order to illustrate the control effect of the semi active control device. TMDs are relatively easy to implement in new structures and retrofitting existing ones. Not like other passive devices, TMDs do not interfere the vertical and horizontal load paths. TMDs efficiency in suppressing the earthquake-induced vibrations of structures besides wind-induced vibrations, have been proved [43]. However, they need a considerable mass to achieve a sizeable reduction in the response. In this part, the optimum parameters of TMD that result in significant drop in vibrations, have been proved. Not like other passive devices, TMDs do no implement in new structures and retrofitting existing ones. As a component in a larger system which leads to far less storage requirements and much lower evaluation time. As a component in a larger

5. Matrices Reduction

The goal of model reduction approach is generating a low dimensional system with the same response characteristics as the original system which leads to far less storage requirements and much lower evaluation time. As a component in a larger

Table 1. Frequencies of Nosrat jacket in dominant modes of vibration

<table>
<thead>
<tr>
<th>Dominant Modes of Vibration</th>
<th>Mass Participant Ratio in each Direction</th>
<th>Frequency (Hz)</th>
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<tbody>
<tr>
<td>X</td>
<td>0.00004</td>
<td>0.3408</td>
</tr>
<tr>
<td>Y</td>
<td>0.84815</td>
<td></td>
</tr>
</tbody>
</table>

\[
\mu = \frac{m}{\bar{\Phi}^\top \mathbf{M} \bar{\Phi}}
\]

Where in Eq.(1), \(m\) stands for the TMD’s mass, \([\mathbf{M}]\) is the platform’s mass matrix and \(\bar{\Phi}\) is the fundamental mode shape normalized to have a unit participation factor. The mass ratio is assumed to be 0.05 and the tuning ratio \(f\) (the ratio of fundamental frequency of the TMD \(\omega_t\) to that of the platform \(\omega_0\)) will be obtained using Eq. (2) [44].

\[
f = \frac{1}{1 + \mu \beta} \left[ \frac{1}{1 + \mu \beta} \right] \left[ \frac{\beta}{1 + \mu} + \frac{\mu}{1 + \mu} \right]
\]

Assuming \(\mu = 0.05\) and \(\beta = 0.02\), TMD mass, tuning ratio and damping ratio will be obtained as: \(m = 210\) ton, \(f = 0.94\), \(\xi = 0.32\). In this study the SATMD is located in main deck elevation as presented in Figure 3 schematically.
simulation or it might be used to develop a low dimensional controller suitable for real-time applications, the resulting reduced model might be used to replace the original system. In other word, Model reduction is a part of dynamic analysis, testing planning, and the control design of structures. Typically, a model with a large number of degrees of freedom, such as one developed for static analysis, causes numerical difficulties in dynamic analysis and high computational cost.

Finally, in structural control design—as used in this study—the complexity and performance of a model-based controller depends on the order of the structural model. In all cases the reduction is a crucial part of the analysis and design. Thus, the reduced-order system solves the above problems if it acquires the essential properties of the full-order model [45, 46].

The modal representation of the jacket has specific controllability and observability properties and its grammians are of specific form. A reduced-order model obtained by evaluating the modal states and truncating the least important. Since the modes with the smallest norm are the last ones in the state vector, a reduced-order model is obtained here by truncating the last states in the modal vector. Modal reduction by truncation of stable models always produces a stable reduced model. The finite time and frequency limited grammians are used in model reduction such that the response of the reduced system fits the response of the full system in the prescribed time and frequency intervals. For model reduction of Nosrat jacket in the finite-time and frequency intervals, the computational procedure can be set up alternatively, either by first applying frequency and then time transformation of grammians, or by first applying time and then frequency transformation which the framework is presented as follows:

Generating the Nosrat jacket type offshore platform and extracting the mass and stiffness matrices, the stationary grammians of controllability (\(W_C\)) and observability (\(W_0\)) of the jacket should be determined utilizing Eq.4 for a given \((A, B, C)\), first. In the second step, the \(W_C(\omega_i)\) and \(W_0(\omega_i)\), \(i = 1, 2\), from Eq.5 is determined, then \(W_C(t_1, \omega_j)\) and \(W_0(t_1, \omega_j)\), \(i = 1, 2, \quad \omega_j = 1, 2\), from Eq.6 will be settled. \(W_C(T, \omega_j)\) and \(W_0(T, \omega_j)\), \(i = 1, 2, \quad \omega_j = 1, 2\) from Eq.7 and Eq.8 will be generated afterwards, and the grammians \(W_C(T, \Omega)\) and \(W_0(T, \Omega)\) over the finite-time interval \(T\) and finite-frequency interval \(\Omega\) (\(T = [t_1, t_2]\), \(t_2 > t_1 \geq 0\), \(\Omega = [\omega_1, \omega_2]\), and \(\omega_2 > \omega_1 \geq 0\)) for the jacket model is determined employing Eq.9. The last step will be applying the reduction procedure to obtain the reduced state-space triple \((A_r, B_r, C_r)\) using grammians \(W_C(T, \Omega)\) and \(W_0(T, \Omega)\) for Nosrat jacket mass and stiffness matrices.

\[ A^T W_C + W_C A + C^T C = 0 \]

\[ W_C(\omega) = W_C S^*(\omega) + S(\omega)W_C \]

\[ W_0(\omega) = S^*(\omega)W_0 + W_0 S(\omega) \]

\[ W_C(t, \omega) = S(t)W_C S^T(t) \]
\[ = W_C(t)S^*(\omega) + S(\omega)W_C(t) \]

\[ W_0(t, \omega) = S^T(t)W_0(\omega)S(t) \]
\[ = W_0(t)S(\omega) + S^*(\omega)W_0(t) \]

\[ W_C(\omega) = W_C(t_1, \omega) - W_C(t_2, \omega), \]
\[ W_C(t, \Omega) = W_C(t, \omega_2) - W_C(t, \omega_1) \]

\[ W_0(t_1, \omega_1) - W_0(t_2, \omega_2) \]

\[ W_C(T, \Omega) = W_C(T, \omega_2) - W_C(T, \omega_1) \]
\[ = W_C(t_1, t_2, \omega_1) - W_C(t_2, t_1, \omega_2) \]

\[ W_0(T, \omega_2) - W_0(T, \omega_1) \]
\[ = W_0(t_1, \Omega) - W_0(t_2, \Omega) \]

As mentioned, the platform model has too many DOFs to be used for the real-time control computations. Belonged to a huge structure, mass and stiffness matrices were large-scaled. Hence, the model has to be simplified with much less DOFs having the same characteristics. As a regard, in order to decrease the time of control calculations, a Matlab function employing the Hankel singular values—derived from two Lyapunov equations and above instructions [46] in finite dimensions—has been generated. As a final point, the 198 DOFs model with modal coordinates in the finite-time and frequency intervals has been reduced to a 25 DOFs model. Figure 4 shows the Hankel singular value diagram.

\[ A^T W_C + W_C A + C^T C = 0 \]  
\[ W_C(\omega) = W_C S^*(\omega) + S(\omega)W_C \]  
\[ W_C(t, \omega) = S(t)W_C S^T(t) \]  
\[ = W_C(t)S^*(\omega) + S(\omega)W_C(t) \]  
\[ W_0(t, \omega) = S^T(t)W_0(\omega)S(t) \]  
\[ = W_0(t)S(\omega) + S^*(\omega)W_0(t) \]  
\[ W_C(t, \omega) = W_C(t_1, \omega) - W_C(t_2, \omega), \]
\[ W_C(t, \Omega) = W_C(t, \omega_2) - W_C(t, \omega_1) \]  
\[ W_0(t_1, \omega_1) - W_0(t_2, \omega_2) \]  
\[ W_C(T, \Omega) = W_C(T, \omega_2) - W_C(T, \omega_1) \] 
\[ = W_C(t_1, \omega_1) - W_C(t_2, \omega_2) \]  
\[ W_0(T, \omega_2) - W_0(T, \omega_1) \]  
\[ = W_0(t_1, \Omega) - W_0(t_2, \Omega) \]

6. LQG Control Algorithm

LQG (Linear system, Quadratic cost, Gaussian noise) controllers, often used for tracking and disturbance
rejection purposes, can typically meet the conditions in which the positioning and tracking requirements should be satisfied for structures with natural frequencies within the controller bandwidth and within the disturbance spectra. The purpose of LQG optimal control algorithm is to suppress and minimize the structural response and control force of the damper simultaneously. Like most optimal control algorithm, LQG algorithm is based on gain matrix which optimizes the performance index [46]. Employing Matlab software, LQG algorithm was applied to reduced matrices of Nosrat jacket model and input voltages of MR dampers based on Eq. (10) were obtained.

\[ V = V_{\text{max}} H(f_{\text{opt}} - f_{\text{meas}}) f_{\text{meas}} \]  \hspace{1cm} (10)

Where in Eq. (10) \( H \) is the Heavy Side Function, \( f_{\text{opt}} \) is the optimized force and \( f_{\text{meas}} \) is the measured force. A closed-loop was defined so as to compute the MR control forces. Seismic excitations were applied to jacket at bed line elevation in both X and Y axis and the signals taken by sensors installed below upper deck elevation sent to compiler. Sensors feedback responses obtained from compiler - applying LQG control algorithm - sent continues signals to MR damper which led to MR damper and finally SATMD controller characteristics adjustment in order to mitigate the dynamic response of the jacket.

7. Nosrat Jacket Acceleration and Displacement Responses

Three different ground motions applied to Nosrat jacket in cases with and without SATMD in order to evaluate the efficiency of semi active control approach. Figures 5-16 presented the results for acceleration and displacement of jacket under Tabas 1978, Bam 2005 Kobe 1995 records. In figures 5-10, the horizontal axis represents the record time in seconds and the vertical axis represents jacket upper deck level acceleration due to excitation, in meters per second squared.

Figures 11-16 indicate Nosrat upper deck level acceleration owing to seismic excitations. In these figures, the horizontal axis represents the record time in seconds and the vertical axis shows upper deck level displacement due to seismic excitation in meters.
The selected DOFs are representative points belonged to sensitive and vital places of the jacket. It can be seen that efficiency of SATMD is high and can suppress the responses obviously. As it is shown, the main deck acceleration employing SATMD reduced from 33.3% to 66.7% due to Kobe earthquake, 44% to 83.3% due to Bam Earthquake and experienced a drop of 25% to 80% due to Tabas earthquake. Furthermore, main/upper deck displacements reductions are as follow: 46.7% to 83.3% due to Kobe earthquake, 29.2% to 78% due to Bam earthquake and 26.7% to 73.7% due to Tabas seismic excitations. However, displacement results in Kobe illustrate increases in first excitation seconds which is justified due to the fact that in these seconds transient responses dominate. Another reason for this increase could be the change occurred in the natural
frequency of the jacket due to adding SATMD [10, 47].
It give the impression that uncontrolled results are acceptable in some cases and there would be no need to add SATMD; however it should be notified that not only the jacket structure but also topside devices’ dynamic responses should be suppressed. According to the sensitivity of topside devices such as production modules, storage units, wellhead modules, flare boom,…to acceleration and displacement and risk of fire and explosion which leads to threaten the safety and serviceability of working crew and devices; employing SATMD is beneficial.

8. Conclusion
Stability importance on one hand and vast expenses of offshore construction besides environmental detrimental effects due to destruction of offshore platform on the other hand, lead to adopt a policy so as to lessen the destruction probability of such structures subjected to operational and environmental loads.
Jacket type offshore platforms, located in seismic-infested area are prone to seismic-induced vibrations. The vibrations may badly threaten the safety of the platform structure and drilling facilities on it; it may also cause great unease of the staff working on the platform. For such structures, it is hard to eliminate vibration entirely. Motion-based design strategy, which permits vibration to exist but seeks to reduce the level of vibration to acceptable ranges so as to specific design objectives, is recognized. In order to suppressing seismic-induced vibrations for jacket type offshore platforms, structural control strategies are discussed.
In this study, a semi active tuned mass damper was proposed in order to reduce seismic-induced vibration of Nosrat jacket type offshore platform. Full scale measurements on Nosrat jacket show that the deck acceleration is consistent and considerable due to the topside equipment which are sensitive to displacement and acceleration. The conceptual model of a SATMD device was presented. To briefly assess the reduction effectiveness of SATMD, numerical feasibility study was conducted on Nosrat jacket type offshore platform in the Persian Gulf under seismic loads. The performance of Nosrat jacket type offshore platform subjected to seismic excitations utilizing SATMD was studied. The time history responses of the platform with and without SATMD have been compared under three different ground motions. The results demonstrate that SATMD can dramatically reduce the acceleration and displacement of the platform. Generally, the results obtained in this study reveal that SATMD, which is commonly used to mitigate wind-induced vibrations in civil engineering, offers substantial performance in mitigating seismic-induced vibrations in offshore platforms. Particularly, for the exploration of marginal oil fields in seismic area, some newly built offshore structures can utilize the control method. By introducing this modification at the design stage, such platforms are more flexible than existing economical jacket offshore structures. Further work needs to be done on the evaluation system of the effectiveness and reliability of the mitigation device.
According to the results and vast expenses of repairing offshore jackets subjected to seismic damages, retrofitting the existing jackets using SATMD is beneficial.

**List of Symbols**

- $(A,B,C)$: triple of the system state-space representation
- $(A_r,B_r,C_r)$: reduced state-space triple
- $A^T$: transpose of matrix $A$
- $A^*$: complex-conjugate transpose of matrix $A$
- $f$: tuning ratio
- $f_{opt}$: optimized force
- $f_{meas}$: measured force
- $H$: Hankel matrix
- $m$: TMD mass
- $M$: mass matrix
- $S$: number of candidate actuator locations
- $V$: MR voltage
- $W_c$: controllability grammian
- $W_o$: observability grammian
- $\beta$: structure’s damping ratio
- $\mu$: mass ratio
- $\omega_i$: $i$th natural frequency
- $\Omega$: matrix of natural frequencies
- $\xi$: damping ratio
- $\phi$: structure’s mode shape

**References**


38- API. (2000), Recommended Practice For Designing and Constructing Fixed Offshore Platforms-Working Stress Design, American Petroleum Institute, USA.


48- IOOC, Inplace Analysis of Platform(DP-A), SD-EST-CN-DPAX-1000-D0, 2000.