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A Numerical Investigation into the Crack Effects on the Natural Frequencies of the Plates

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ABSTRACT

In the current study, the effects of some parameters such as length, position and angle of the cracks on the free vibrations of a plate are examined with the aid of the finite element method. While the crack was assumed to be internal and through, its growth was ignored. The crack was modeled as a discontinuity in the plate model. The results show that the thickness and mechanical properties of the plate do not affect the ratio of natural frequency of the cracked plate to the natural frequency of its corresponding intact plate. However, other crack parameters influence on the stiffness and natural frequencies of a plate through affecting its modal shapes and stress intensity factors.

1. Introduction

Nowadays, plates are extensively used in various engineering fields such as aerospace, civil, mechanical and marine structures. Study of the dynamics for the plate structures aiming at realizing their optimum performance in vibrating media is not only useful but also necessary. In order to achieve this aim, important properties of a structure like natural frequencies and mode shapes should be provided and the factors affecting them should be determined. Crack is one of the important shortcomings of plate structures, which influences their dynamics. Hence, the analysis of the effect of cracks on free flexural vibrations of thin plates is crucially important for engineers and designers.

Vibration problem of a plate is analytically solved in two steps: the choice of the theory and selection of the solving method. The prevailing plate theories are Kirchhoff's classical thin plate theory (CPT) [1], Mindlin's first-order shear deformation plate theory (FSD) [2], Reddy's higher-order shear deformation plate theory (HSDT) [3], and 3-D theory of elasticity [4]. Solving methods are generally divided into two categories of precise and approximate methods. Out of a wide range of plates and shells, only a limited number can be precisely analyzed owing to their geometry and boundary conditions. Other plates can be analyzed only by approximate methods. Approximate methods are by themselves divided into two types: the first type is to use the method of minimizing structure energy such as variational integral method, Galerkin's method and Rayleigh-Ritz's method; and the second type is finite difference and finite element methods.

Depending on the applied theory, the equations of motion of plates and shells can be derived by classical (Newtonian) or energy (Hamilton) method which is arbitrary since they lead to identical partial derivative equations if mathematical operations are conducted correctly. However, energy methods are necessary to be used for deriving equations of motion when such methods as Extended Kantorovich's or Rayleigh-Ritz's methods are used.

Owing to the computational complexities of precise vibrational analysis and even approximate first-order analysis of cracked plates, limited studies have been so far conducted in this field mostly considering simply supported plates on four edges or two opposite edges. This is because there are only precise analytical solutions for this kind of plates in non-cracked state and also quasi-analytical solutions for them in cracked state having either parallel or vertical cracks.

So far, only Hosseini-Hashemi et al. [5] in their study in 2010 have offered explicit, precise characteristics equations for analyzing free vibrations of moderately thick plates with arbitrary number of all-over part-through cracks. In 2011, Huang et al. [6] used Ritz method for determining natural frequencies and mode shapes of thick, cracked rectangular plates and considered two types of crack arrangements, namely side crack and internal crack. In 2009, Israr et al. [7] proposed an analytical model for vibrations with imposing external load on cracked rectangular plates with different boundary conditions and analyzed the equations by Galerkin's method. They considered cracks as parallel to plate edges, too. In 2009, Huang and Leissa [8] examined the analysis of the vibrations of rectangular plates with side cracks by Ritz's method. They first inferred energy function of plate vibrations and then, minimized energy function by introducing a new, complete function for transverse displacement of the plate in order to be able to calculate natural frequencies. In 1994, Liew [9] and in 1993, Kee et al. [10] used a so-called domain or sub-domain decomposition method for determining natural frequencies of a cracked plate. The advantage of this method is that there is no need to special relations for including singularity stress in crack tip. In 1983 and 1985, Solecki [11, 12] analyzed flexural vibrations of a thoroughly cracked plate and applied Fourier transformation functions to the governing equation to specify an integral equation system which included the unknowns of displacement and the slope of crack planes. The coefficients of unknowns are determined by applying the conditions in which the flexural and shearing forces working on crack planes equal zero. This method is only used for simply supported conditions. In 1972, Stahl and Keerd [13] suggested equations for determining natural frequencies of plates with a thin crack emanating from on edge and with a centrally located internal crack.

All foregoing methods are just applicable for special cases of vibrational aspects of plates. The real engineering structures are much more complicated which necessitates the application of numerical methods such as the finite element method.

In 2009, Bachene et al. [14] used extended finite element method for examining the vibrations of a plate. They studied two types of cracks, namely central and through-edge cracks. The applied elements were four-nodal and nine-nodal and the numerical calculations were carried out in FORTRAN.

In 2001, Krawczuk et al. [15] suggested a finite element model for a plate with elasto-plastic through crack which included crack tip plasticity, too. The calculations were conducted in both cases of completely plastic and completely elastic cracks. They found that crack tip plasticity increased the effect of crack on natural frequencies of the plate. However, the difference between the results for completely elastic crack and those for completely plastic crack was very slight which can be neglected in engineering applications.

In 1994, Krawczuk and Ostachowicz [16] and in 1991, Qianet al. [17] proposed a finite element model for a cracked plate and applied it in vibration analysis. They first estimated the stress intensity factor of a finite element with a through crack under bending, twisting and shearing and then, they calculated stiffness matrix of a finite element of a cracked plate using stress intensity factor by calculating flexibility matrix in which the flexibility matrix of a cracked plate finite element was assumed to be the sum of the flexibility matrix of the non-cracked finite element and an additional flexibility matrix caused by the crack. Stress intensity factor has been studied in crack mechanics in detail. Nonetheless, there is still no precise solution for calculating stress intensity factors in modes I, II and III of the crack for a finite rectangular plate with a through crack. This great drawback can be overcome for dynamics problems by the method proposed in these papers because stress intensity factor for finite plates with through crack has been approximately estimated only for modes I and II of the crack mechanics.

The main objective of the current paper was to develop a thorough finite element model for thin cracked plates by taking all parameters affecting natural frequencies into account. These parameters included boundary conditions, mechanical properties and geometry of the plate, together with the length, position and angle of the crack. After detailed modeling, the vibrations of the cracked plates were studied.

2. Finite Element Equations

In the present paper, quadrilateral plate bending element based on discrete Kirchhoff theory (DKT) was used for forming structural stiffness matrix. This element has one translational degree of freedom and two rotational degrees of freedom at each of its nodes. The applied equations relevant to a triangular plate bending element based on discrete Kirchhoff theory (Figure 1) are presented in Figure 2. A quadrilateral element is formed by triangular DKT elements. As can be seen, the quadrilateral is divided into two triangles and then, two further triangles are added to it. Consequently, there are two diagonals which are formed by four triangular elements. Therefore, the resulting stiffness is divided by 2. The equations related to stiffness matrix of DKT triangular element are presented in [4] and [18].

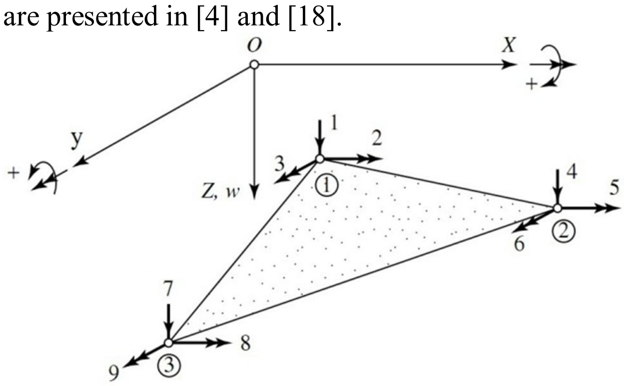
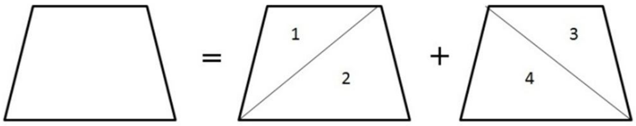


Figure 1. A triangular shell element



$$k = \frac{1}{2}\left(k_1 + k_2 + k_3 + k_4\right)$$

Figure. 2. Generation of a quadrilateral shell element using triangular shell elements

2.1. Mass Matrix

When implementing the finite element method for dynamics problems of a plate, mass matrix exhibits the inertial properties of the plate structure. Mass is a continuous, distributed property in plate structure. So, if a discontinuous method like FEM is used, the continuous mass of the plate should be discretized too. There are two methods for the calculation of mass matrix of a plate in the finite element method: lumped mass matrix and kinematically consistent mass matrix. In finite element equations for free vibrations of the plates, the precision of eigenvalues is improved particularly in higher modes by using a mass matrix with constant kinetic energy instead of lumped mass matrix. Such a matrix can be obtained by comparing kinetic energy of the main structure with that of decomposed system of finite elements. Therefore, the mass matrix by the foregoing methods would be as follows [4]:

$$M_e = \mu \int_0^a \int_0^b N_i^T N_j dx\ dy \tag{1}$$

The shape functions, N_i and N_j, are the same functions that are used for calculating bending stiffness matrix coefficients. It should be noted that the mass matrices of a cracked plate and of a non-cracked plate are equal and that the crack only influences stiffness matrix [16, 17].

2.2. Finite element equation of motion

In problems which include linear elastic materials obeying the Hooke's law, the governing equations is a set of mass and spring equations and in total, the equations of motion would take the following form [18, 19]:

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = \{F(t)\} \tag{2}$$

Where $[M]$ is mass matrix, $\{\ddot{U}\}$ is acceleration vector, $[C]$ is damping matrix, $\{\dot{U}\}$ is velocity vector, $[K]$ is stiffness matrix, $\{U\}$ is displacement vector, and $\{F(t)\}$ is force vector. In modal analysis, damping is excluded. Then, the following equation is resulted:

$$[M]\{\ddot{U}\} + [K]\{U\} = 0 \tag{3}$$

Supposing harmonic movement in free vibrations solution, this equation is reduced to the following eigenvalues equation:

$$[M]\{U\}\lambda + [K]\{U\} = 0 \tag{4}$$

where, λ is a eigenvalue equal to ω^2.

2.3. Stress intensity factor effect

The first experiments on the cracks in a glass environment were carried out by Griffith whose results were published by the Journal of Royal Society in 1928 [20]. The so-called Griffith method is still valid even considering the recent advances in laboratorial equipment, since the simplifying assumptions for the behavior of materials are not considered in this method. The energy relation according to Griffith theory is as follows ignoring plastic energy and kinetic energy for a growthless, simply supported crack [20]:

$$U_E = U_i - U_a \tag{5}$$

Where, U_E is elastic strain energy of a cracked plate, U_i is elastic strain energy of a non-cracked plate, and U_a is the elastic strain energy released by the crack. Given this relation, the crack decreases elastic strain energy and stiffness of the plate.
Supposing plane stress condition, the released elastic strain energy in terms of stress intensity factors is as follows [20]:

$$U_a = \frac{1}{E}\int\left(K_I^2 + K_{II}^2 + (1+\nu)K_{III}^2\right)dA \tag{6}$$

Given eq. (6), it can be concluded that as stress intensity factor is increased, the elastic strain energy released due to the crack is increased and that as elastic strain energy of a cracked plate (U_E) is decreased, plate stiffness is decreased. Therefore, it can be inferred that the stiffness of a plate is decreased (increased) with the increase (decrease) in stress intensity factor.
The objective of the current paper is not to calculate stress intensity factors, but rather to insist that they can be compared with each other using the foregoing relations provided that the vibrational mode shapes are in hand. Thus, the extent of the effect of stress intensity factors on the natural frequencies can be measured qualitatively and not quantitatively.
In order to make a comparison among the values of the stress intensity factors, the judgments must be based on the parameters affecting them which include crack length and the stresses on crack plane [20]:

$$K_i = \sigma_i \sqrt{\pi c}\ ,\ i=I, II, III \tag{7}$$

Another simple, appropriate criterion is the displacements of crack tip in three directions of the coordinates, because stress intensity factors are proportional to these displacements by the following relations [5]:

$$K_I = \sqrt{2\pi}\,\frac{2G}{1+k}\left|\Delta V\right| \propto \left|\Delta V\right| \tag{8}$$

$$K_{II} = \sqrt{2\pi}\,\frac{2G}{1+k}\left|\Delta U\right| \propto \left|\Delta U\right| \tag{9}$$

$$K_{III} = \sqrt{2\pi}\,2G\left|\Delta W\right| \propto \left|\Delta W\right| \tag{10}$$

In plane stress condition, it can be written that $k = \dfrac{3\nu}{1+\nu}$.

2.4. Finite element model for the cracked plate

In total, finite element modeling of the cracked plates can be achieved by three methods:

1. Common finite element method, in which the crack is modeled by separating the finite elements along transverse planes of the crack. This method needs a very fine mesh around crack tip due to the singularity properties of the stress and strain in this region. As shown in Figure 4, the elements separated along transverse planes can be contact elements, while the triangular elements can be used instead of rectangular elements in crack tip for modeling the stress intensity. Such a model is used in the fracture mechanics analyses for the calculation of stress intensity factors in the crack tip regions where singularity stresses occur and also for the calculation of buckling where crack edges are compressed.
2. In this category, the crack is modeled by two elements. This method needs a lower mesh density. As shown in Figure 5, singularity properties of the stress and strain in crack tip are not included in this kind of modeling. This model is used for the calculation of buckling and vibrations because other singularity properties of stress and strain in crack tip are not important.
3. In the third method, crack is modeled by one element resulting in a great saving of required time for both modeling and calculations compared to other methods (Figure 6). This method can be used for modal analysis of the cracked plates, where the out-of-plane displacements of the plate are studied, while the effect of contact between the crack planes is negligible [21] and the singularity properties of stress and strain is immaterial. This method greatly saves calculation time. Nonetheless, when in-plane vibration of plate is studied, the contact between crack planes has a considerably influence and the models represented in Figure 4 or Figure 5 must be applied.

As shown in Figure 7, the crack of a plate is considered as to be discontinuous, so that two straight lines overlap. The applied method is based on separation of finite element nodes along crack planes and as mentioned above, in this case there is no need for thinner meshing around crack tip. A sample node numbering scheme for the lines including crack is represented in Figure 7.

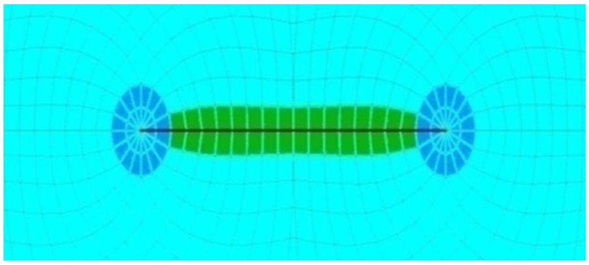


Figure 4. Common finite element modeling of crack

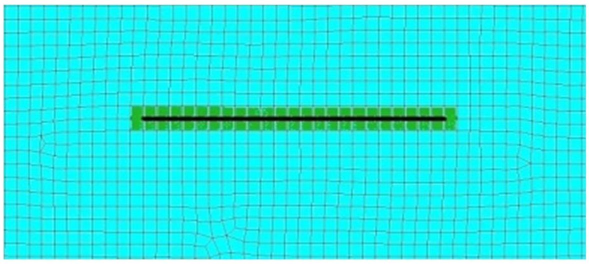


Figure 5. Use of two elements in crack modeling

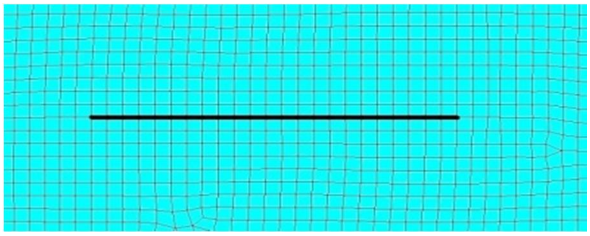


Figure 6. Use of one element in crack modeling

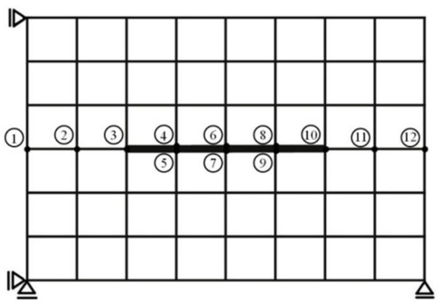


Figure 7. Crack line node numbering

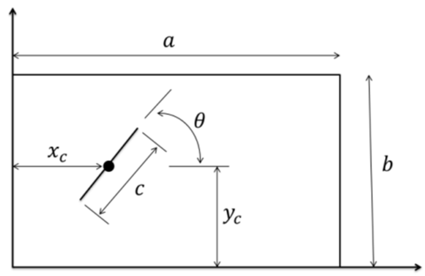


Figure 8. Geometric parameters for the plate and crack

In the current study, a thorough home program of FEM was developed and used for the analysis of the cracked plate models. This program reads the input data of the model for analysis and takes all the required steps for pre-analysis, analysis and post-analysis including the definition of geometry, meshing, the application of boundary conditions, modal analysis and finally, post-analysis activities. The plate model (Figure 8) has variable material and geometry properties, while the length, angle and position of the crack also may be changed.

To validate the results of the present modeling scheme, they were compared with the results of two examples explained in the pre-referred papers. The results were expressed in terms of the ratio of natural frequencies of the cracked plate to those of the corresponding intact plate.

The first example deals with a square plate simply supported on four edges with the specifications shown in Figure 9 and Table 1. For different relative lengths of the crack, Tables 2 and 3 compare the results of the applied model of the cracked plate in this study with those obtained by the previous researchers. The comparisons reveal that the present model has an acceptable level of accuracy. As said in the section 'Introduction', the results of Solecki [12] were obtained from analytical method, while those in the references [16] and [17]are based on the finite element method. The advantage of the method applied in this paper over the methods applied in [16] and [17] that are similar, is that there is no need to special relations for including the singularity stress on the crack tip.

The second validation example is a plate with one clamped edge, which its properties are given in Table 4 and Figure 10. The obtained results for such a case are compared against the experimental results.

As can be seen in Table 5, the experimental results and the results of the model applied in this study had negligible differences in the first and second modes, but the difference grew in the higher modes. The difference or discrepancy depends on two factors: idealization in theoretical relations and experimental error because in theoretical modal analysis, it is assumed that damping coefficient is null and the materials are completely elastic, but these assumptions do not hold for the experiment.

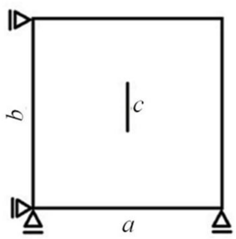


Figure 9. Simply-supported cracked plate

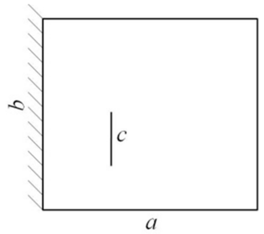


Figure 10. Cracked plate with one clamped edge

Table 1. Properties of the simply-supported cracked plate

E	ρ	ν	a	b	h	θ	$\frac{x_c}{b}$	$\frac{y_c}{a}$
204×10^9	7860	0.3	0.1	0.1	0.001	90	0.5	0.5

Table 2. Comparison of obtained natural frequency ratios for the simply-supported cracked plate with the previous results

$\frac{\omega_{cp}}{\omega}$	1st mode				2nd mode			
c/a	[12] Analytical	[16] FEM	[17] FEM	Present FEM	[12] Analytical	[16] FEM	[17] FEM	Present FEM
0.05	0.9970	0.9971	0.9975	0.9992	0.9999	1.000	0.9999	1.000
0.1	0.9940	0.9942	0.9950	0.9957	0.9998	1.000	0.9999	0.9998
0.15	0.9855	0.9874	0.9885	0.9895	0.9982	0.9980	0.9989	0.9991
0.2	0.9775	0.9806	0.9820	0.9810	0.9966	0.9970	0.9980	0.9970
0.25	0.9659	0.9682	-	0.9702	0.9895	-	-	0.9926
0.3	0.9530	0.9548	-	0.9580	0.9824	-	-	0.9842

Table 3. Comparison of obtained natural frequency ratios in 3rd mode of vibration for the simply-supported cracked plate with the previous results

c/a		0.05	0.1	0.15	0.2	0.25	0.3
[16]	FEM	1.000	1.000	0.9999	0.9998	0.9994	0.9990
Present	FEM	1.0000	1.0000	0.9998	0.9996	0.9991	0.9981

Table 4. Properties of the cracked plate with one clamped edge

E	ρ	ν	a	b	h	$\frac{c}{a}$	θ	$\frac{x_c}{b}$	$\frac{y_c}{a}$
67×10^9	2800	0.33	0.24	0.24	0.00275	0.5	90	0.375	0.375

Table 5. Comparison of obtained natural frequency ratios for the plate having one clamped edge with the previous results

	$\frac{\omega_{cp}}{\omega}$	1st mode	2nd mode	3rd mode
[17]	FEM	0.9931	0.9989	0.9837
[17]	Experimental	0.9917	0.9981	0.9807
[14]	Extended FEM	0.9925	0.995	0.9936
Present	Present	0.9952	0.9976	0.9989

3. Influence of Fluctuation of Mechanical Properties and Thickness on Ratio of Natural Frequencies (Ω_{cp}/Ω)

Mechanical properties, i.e. Young's modulus (E), density (ρ) and thickness (h) influence the natural frequencies of an intact plate. Properties of the analyzed models with variable Young's modulus, density and thickness are given in Tables 6 to 8.

Considering Figures 11-16, mechanical properties and thickness do not influence the ratio of natural frequencies of a cracked plate to those of an intact plate under both simply and clamped support conditions. Hence, the effect of the change in the material properties and thickness on natural frequencies of a cracked plate is similar to that of an intact plate [22].

Table 6. Properties of the model with variable Young's modulus

ρ	ν	$\frac{a}{b}$	h	$\frac{c}{a}$	θ	$\frac{x_c}{b}$	$\frac{y_c}{a}$
8000	0.3	1	0.03	0.5	0	0.5	0.5

Table 7. Properties of the model with variable density

E	ν	$\frac{a}{b}$	h	$\frac{c}{a}$	θ	$\frac{x_c}{b}$	$\frac{y_c}{a}$
200×10^9	0.3	1	0.03	0.5	0	0.5	0.5

Table 8. Properties of the model with variable thickness ratio

$\frac{a}{b}$	$\frac{c}{a}$	θ	$\frac{x_c}{b}$	$\frac{y_c}{a}$
1	0.5	0	0.5	0.5

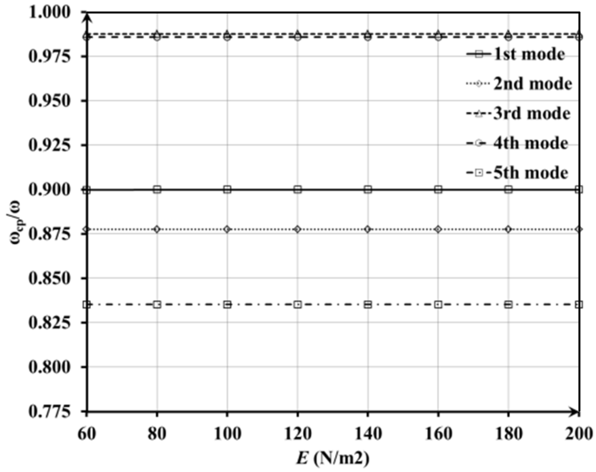


Figure 11. Effect of Young's modulus on the natural frequency ratio of the simply-supported plate

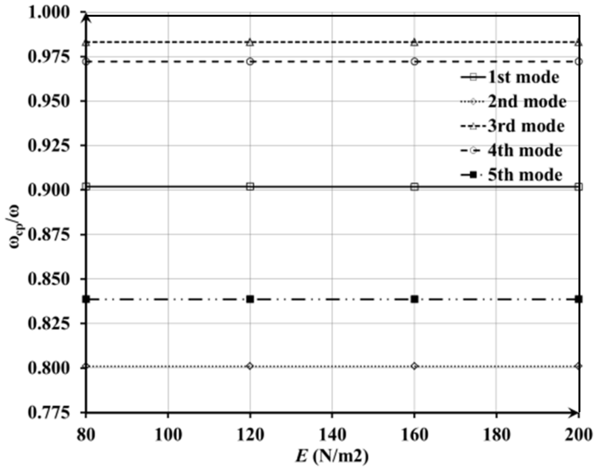


Figure 12. Effect of Young's modulus on the natural frequency ratio of the clamped plate

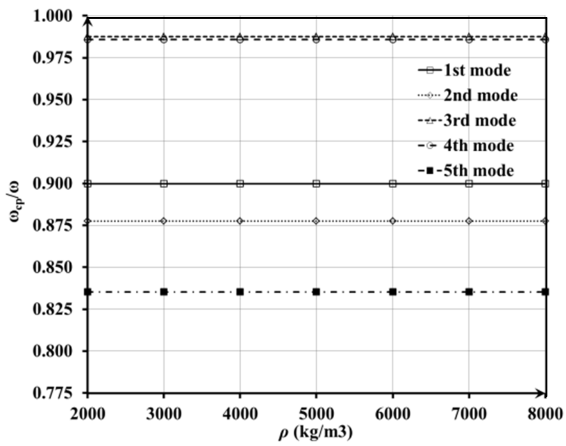


Figure 13. Effect of density on the natural frequency ratio of the simply-supported plate

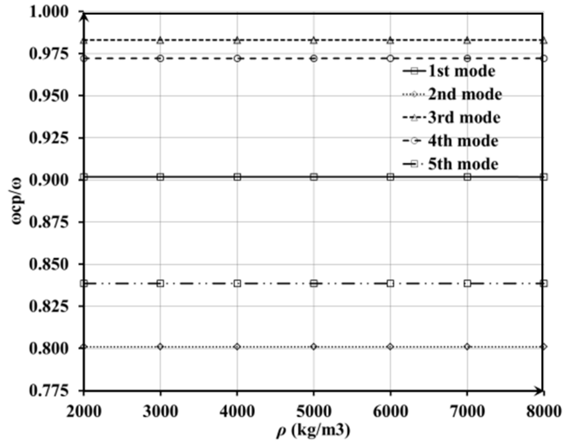


Figure 14. Effect of density on the natural frequency ratio of the clamped plate

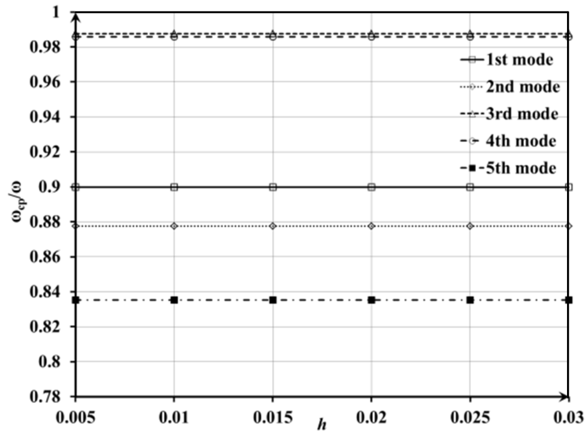


Figure 15. Effect of thickness ratio on the natural frequency ratio of the simply-supported plate

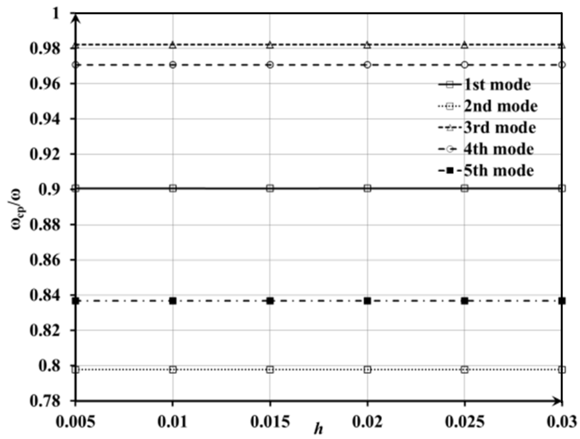


Figure 16. Effect of thickness ratio on the natural frequency ratio of the clamped plate

3.1. Influence of fluctuation of a/b on ratio of natural frequencies (ω_{cp} /ω)

In this section, the vibrations of a rectangular plate with different length-to-width ratios are studied. Although dimensional *a/b* ratios of over 5 are not applicable in industry particularly in shipbuilding industries, they would be considered in order to study the convergence of the ratio of natural frequencies.

According to Figure 17 and Figure 18, as length-to-width ratio fluctuates, the ratio of natural frequencies

alternately fluctuates and the maximum influence of crack is in a different mode for different length-to-width ratios. This is because modal shapes change with the change in length-to-width ratio according to Tables 9 and 10. But, in mode 1 under both support conditions, relative frequencies continuously decrease as *a/b* is increased because mode shape does not change and also, given the increase in the length of plate (a) with the increase in *a/b*, absolute length of crack (c) is increased since the relative length of crack is constant. Subsequently, based on the eq. (7), stress intensity factor increases and plate stiffness decreases. As well, it can be observed that the variations of the ratio of relative natural frequencies differ with changes in the supporting states, namely simply supported and clamped. But in both cases, relative natural frequencies converge with the increase in *a/b*.
It can be inferred from Tables 10 and 11 that in mode 1 under both support states of simple and clamped, cracks do not change the number of longitudinal and transverse half-waves and only influence the maximum and minimum amplitudes. But in higher modes, cracks affect the number of longitudinal and transverse half-waves as well as maximum and minimum amplitudes and they cause a decrease in the natural frequencies not only by influencing the stiffness of the plate but also by affecting modal shapes.

Table 9. Properties of the model with variable length-to-width ratio

h	$\frac{c}{a}$	θ	$\frac{x_c}{b}$	$\frac{y_c}{a}$
0.03	0.5	0	0.5	0.5

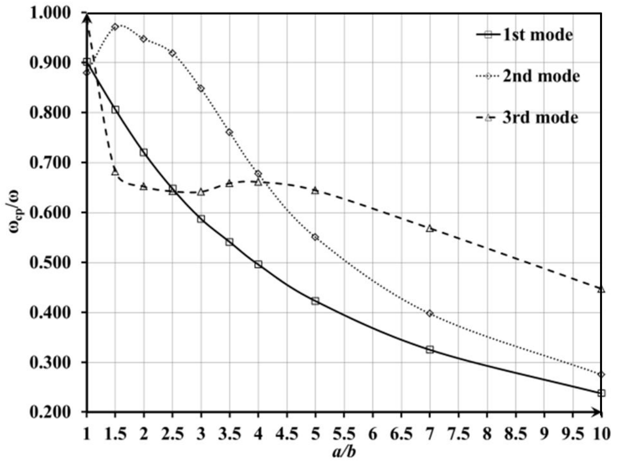


Figure 17. Effect of length-to-width ratio on the natural frequency ratio of the simply-supported plate

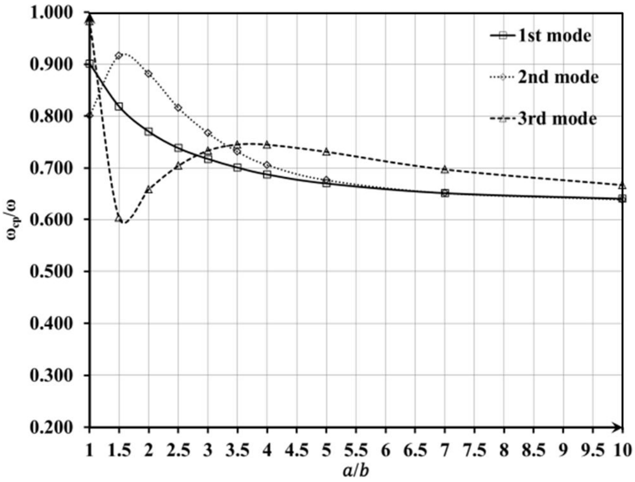
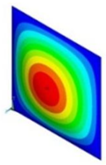
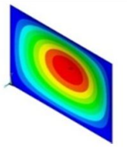
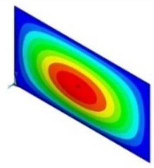
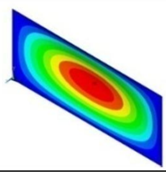
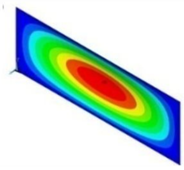
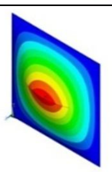
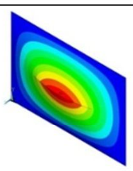
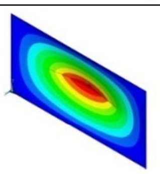
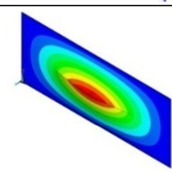
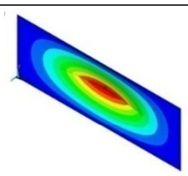
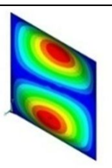
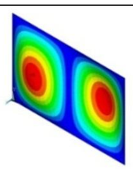
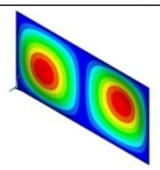
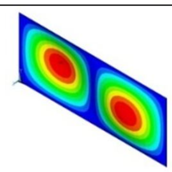
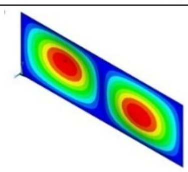
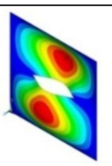
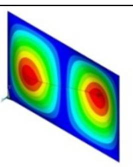
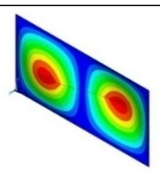
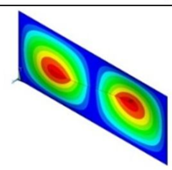
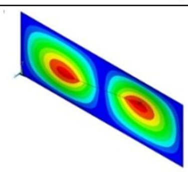


Figure 18. Effect of length-to-width ratio on the natural frequency ratio of the clamped plate

Table 10. Changes in the mode shape of the simply-supported plate as a result of changing the length-to-width ratio

		$\gamma = 1$	$\gamma = 1.5$	$\gamma = 2$	$\gamma = 2.5$	$\gamma = 3$
1st mode	Intact					
	Cracked					
2nd mode	Intact					
	Cracked					

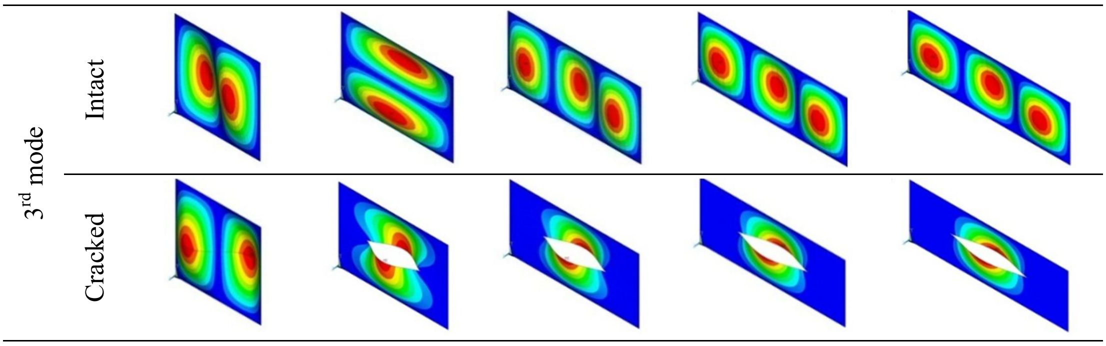
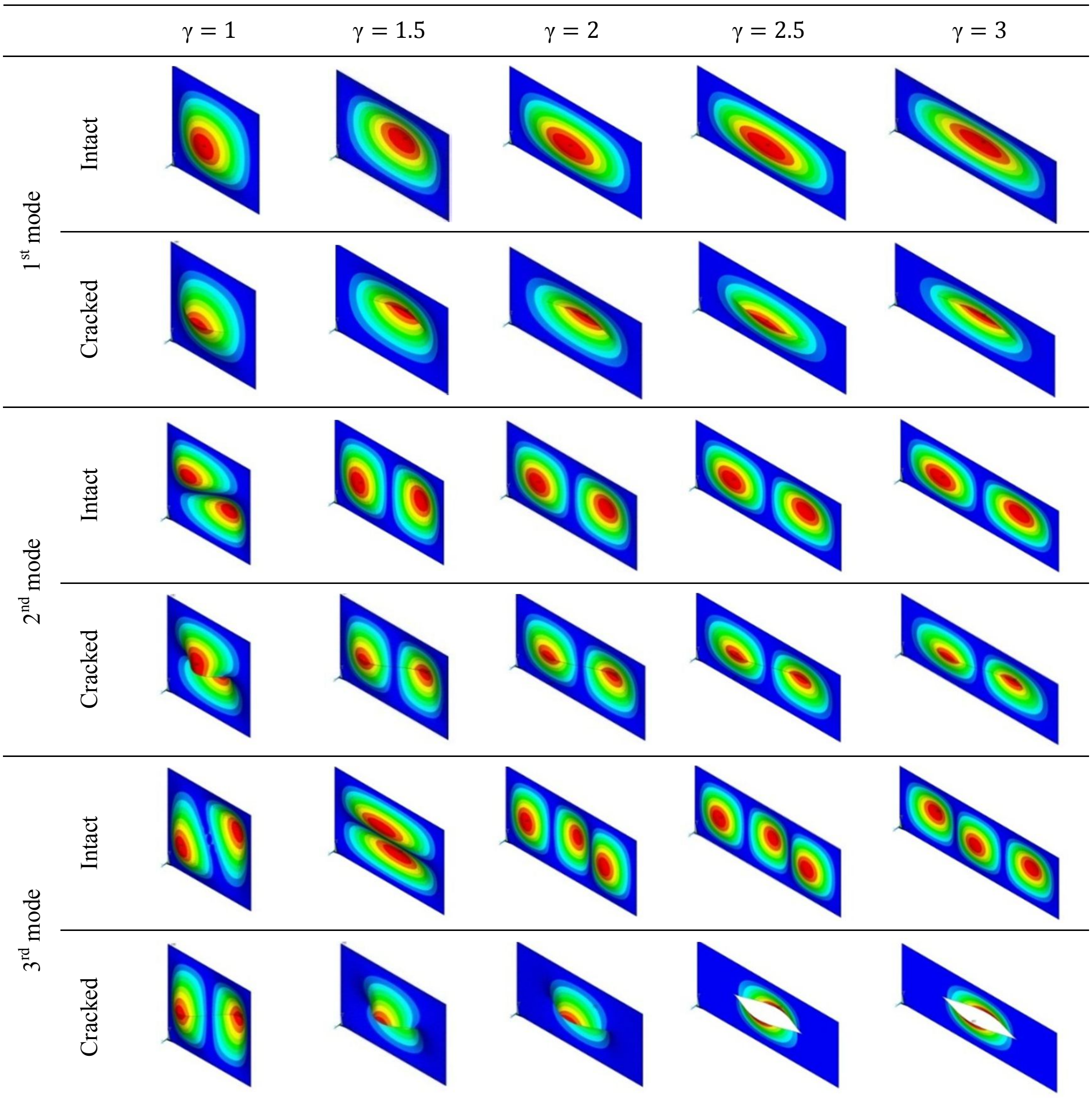


Table 11. Changes in the mode shape of the clamped plate as a result of changing the length-to-width ratio



In fact, to understand the behavior of crack in plate it should be studied that which of the sources, stress intensity factor (plate stiffness) or the change of modal shapes, is the prevailing cause of the changes in the values of the ratio of natural frequencies as caused by the variations of *a/b*.

In the second mode under both support states of simple and clamped, according to the relevant Figs. and Tables, when the form of the plate is changed from square to rectangular, the modal shapes of the intact and cracked plates change. This in turn results in higher relative natural frequencies (i.e. the reduction of the effect of crack) in such a mode, since it can be seen that stress intensity factor is higher in square plate than in rectangular plate in proportion to the displacements of crack tip. But, after the transit from square to rectangular state, relative natural frequency starts to continuously decrease according to the relevant graphs, like the first mode, since modal shapes do not change.

The phenomenon which occurs with the transit from square state to rectangular state in the second mode is reversed in the third mode under both support states of simple and fixed. In other words, given the modal

shapes in the third mode and the explanations of the previous paragraph, relative natural frequency decreases with the increase in stress intensity factor and then, the diagram of relative natural frequencies does not follow a regular trend because the modal shapes of intact and cracked plates differ and change.
Compared to other modal shapes, a strange modal shape can be seen in cracked square plate under the second mode and in cracked rectangular plate under the third mode. This modal shape is antisymmetric on x-axis and indeed, it divides the plate into two pieces, induces a high stress intensity factor, greatly influences the reduction of natural frequencies and thus can be regarded as a critical mode.

3.2. Influence of fluctuation of crack relative length (c/a) on ratio of natural frequencies (ω_{cp}/ω)

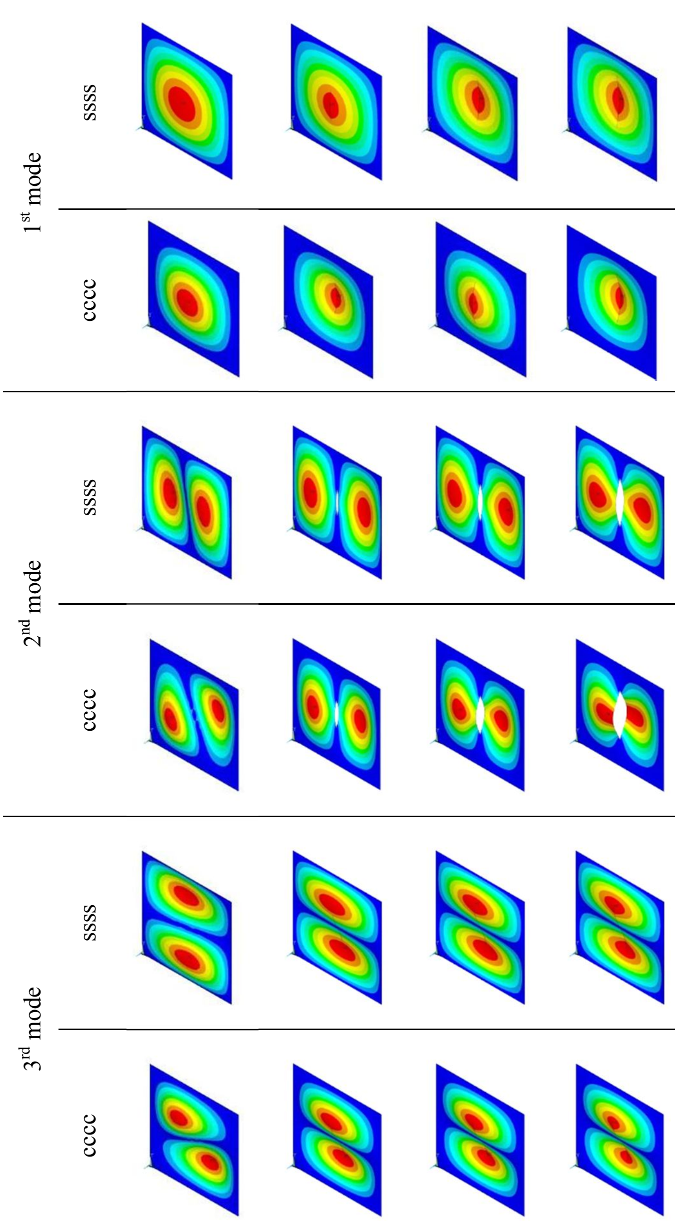
As can be seen in Figures 19 and 20, crack in total results in the decrease in natural frequencies compared to non-cracked state and the greater the crack length, the greater the decrease in the relative frequencies. Because according to eq. (7), the increase in crack length results in the increase in stress intensity factors and the loss of plate stiffness.
The vibrational behavior of a cracked square plate induced by the fluctuation of crack length is similar under clamped support to that under simple support although the numerical values of relative frequencies are different. For example, the influence of crack in the second mode under clamped support state is greater than that under simple support state because given the modal shapes according to Table 13, the deformation of the crack edges in the second mode is greater under fixed support state than under simple support state.
In addition, the influence of crack on different modes varies. For example, given Table 13, crack only slightly influences the third mode under both simple and fixed support states.This is because the form of the modes, i.e. half-waves, is so that the stress imposed on transverse planes of crack are in one direction so that little stress intensity occurs and the deformation of cracked plate compared to intact plate is slight. But in the second mode, the displacements of crack edges are great and subsequently, as stress intensity factor increases its effect on the reduction of relative frequencies increases.

Table 12. Properties of the model with variable crack length

$\frac{a}{b}$	h	θ	$\frac{x_c}{b}$	$\frac{y_c}{a}$
1	0.03	90	0.5	0.5

Table 13. Mode shapes in different crack length ratios

$\frac{c}{a}$	0	0.3	0.4	0.5



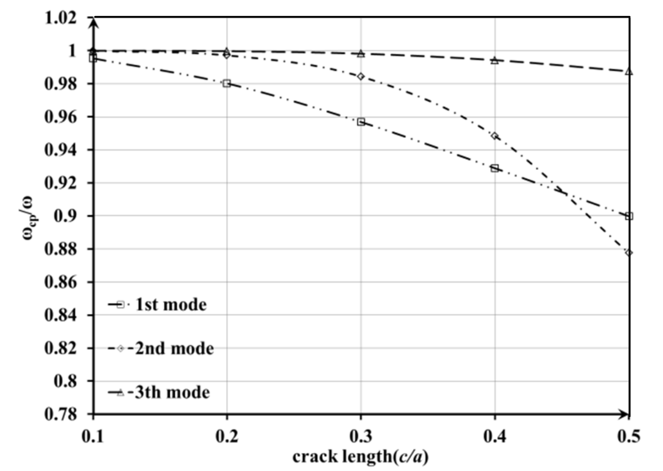


Figure 19. Effect of crack length on the natural frequency ratio of the simply-supported plate

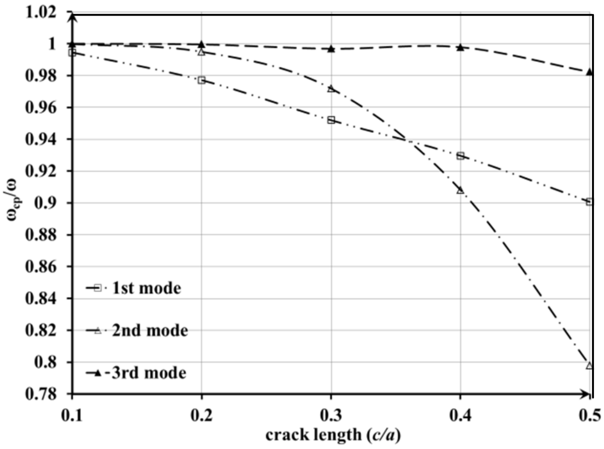


Figure 20. Effect of crack length on the natural frequency ratio of the clamped plate

3.3. Influence of angle change (θ) on ratio of natural frequencies (ω_{cp}/ω)

Also, some models with different angles of orientation for the crack are analyzed, the properties of which are given in Table 14. According to Figures 21 and 22 and also mode shapes shown in Tables 15 and 16, it can be seen that when the plate is not square and the crack is situated in the center of plate, the changes in angle sensibly influence relative frequencies in all three modes because in the first mode under both support states, as angle changes from 0 to 90°, given the parameters affecting stress intensity factor, it reduces on crack tip and plate stiffness increases and subsequently, the influence of crack decreases with the increase in relative frequencies. This is reverse in the case of the second and third modes.

In order to find the causes of the reduction of stress intensity factor in the first mode in Figures 21 and 22, the following relations can be applied on Kirchhoff's plate bending [4]:

$$\sigma_x \propto \left(\frac{1}{\rho_x} + \nu \frac{1}{\rho_y} \right) \tag{11}$$

$$\sigma_y \propto \left(\frac{1}{\rho_y} + \nu \frac{1}{\rho_x} \right) \tag{12}$$

These relations show that the longitudinally and transversely imposed stresses (σ_x, σ_y) are indirectly proportional to the longitudinal and transverse curvature radiuses of the plate (ρ_x, ρ_y) and since the curvature radius of the plate is higher in length than in width, the longitudinal stress is lower than the transverse one too.

When the crack occurs in 0° angle under the first modal shape, σ_y is imposed on crack planes and when it occurs in 90° angle, σ_x is imposed on them. Therefore, given these facts and relation (7), stress intensity factor on crack tip in the first mode in 0° angle is greater than that in 90° angle under both simple and clamped support states because σ_y is greater than σ_x. Also, in the first mode it is clear for square plate that the curvature radius of the plate is equal in length and width. Thus, as crack angle changes from 0° to 90°, the angle of the stress imposed on crack planes and then, the stress intensity factors do not change. So, the change in crack angle does not influence the natural frequencies on square plate.

It was seen in the second mode that as the angle change from 0° to 90°, the displacements of crack tip increase and on the basis of relations (8), (9) and (10), stress intensity factors increase and consequently, the influence of the crack on the reduction of relative natural frequency increases in the second mode too.

Table 14. Properties of the model with variable crack direction angle

$\frac{a}{b}$	h	$\frac{c}{a}$	$\frac{x_c}{b}$	$\frac{y_c}{a}$
2	0.03	0.5	0.5	0.5

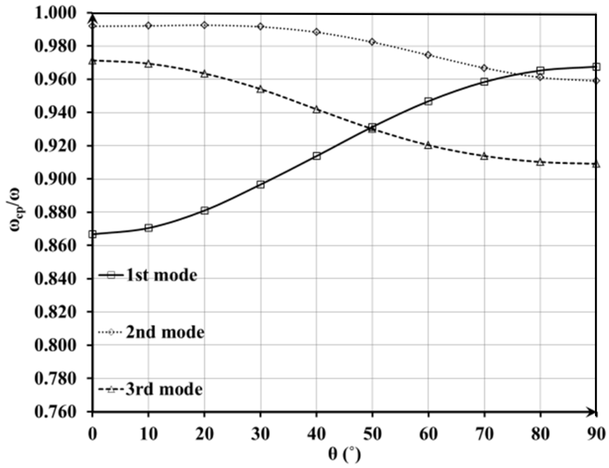


Figure 21. Effect of crack direction angle on the natural frequency ratio of the simply-supported plate

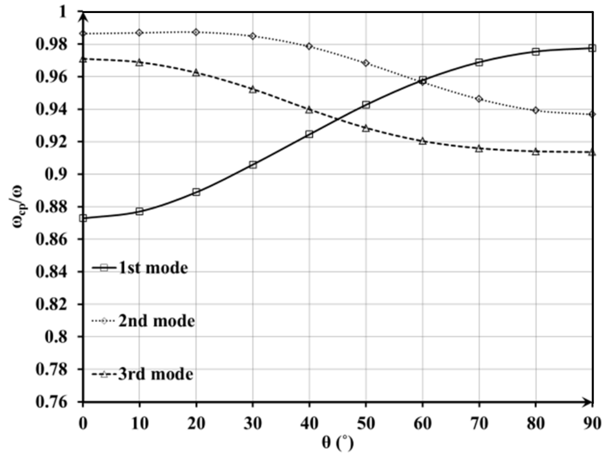


Figure 22. Effect of crack direction angle on the natural frequency ratio of the clamped plate

Table 15. Mode shapes in different crack direction angles in case of simply-supported plate

θ	0	30	60	90

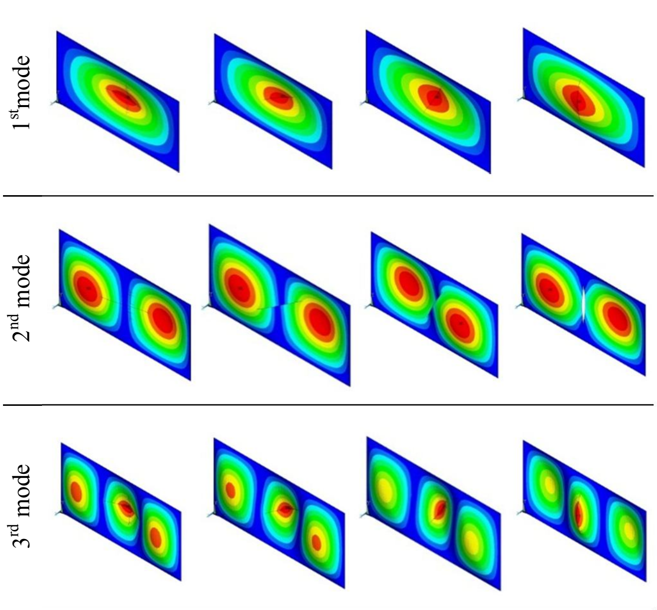
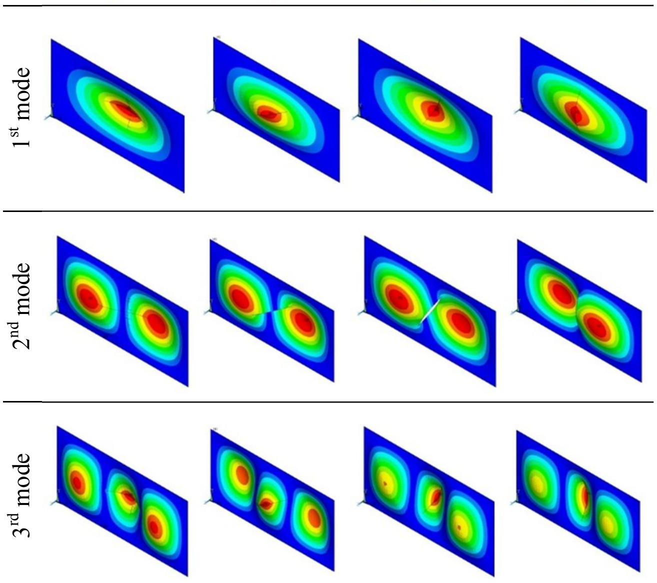


Table 16. Mode shapes in different crack direction angles in case of clamped plate

θ	0	30	60	90



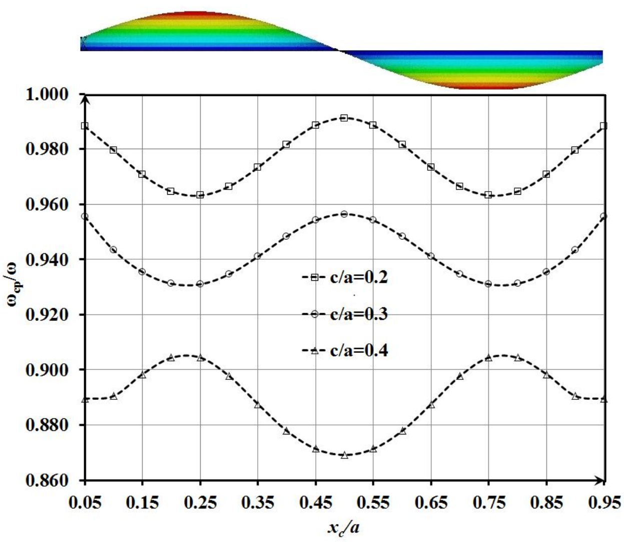
3.4. Influence of displacement of crack center (x_c/a) on ratio of natural frequencies (ω_{cp}/ω)

Nodal line is a line on which the displacements of nodes are zero. The study of the influence of crack on natural frequencies regarding their position in relation to nodal lines can be interesting. Tables 10 and 11 show the nodal lines of a simply and fixed supported intact plate. For example, when the plate is rectangular in the second mode, not square, there are two longitudinal half-waves and one transverse half-wave and the line between two longitudinal half-waves is a nodal line.

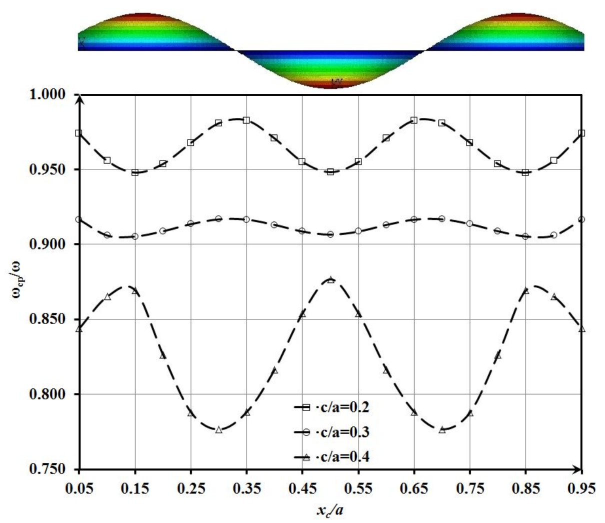
According to the results shown in the Figures 23 and 24 for the plate models with varying position of the center of the crack (Table 17), when relative length of crack (c/a) is small, crack has the minimum effect on the ratio of natural frequencies on nodal line and has the maximum effect on the spacing between two successive nodal lines. But when relative length of crack increase, this influence decreases and in long relative lengths of the crack, it rather reverses. In other words, crack has the maximum effect on the ratio of natural frequencies on nodal line and has the minimum effect on the spacing between two successive nodal lines because when the crack is long, crack planes act like a free edge and displacements in crack edges increase.

Table 17. Properties of the model with variable crack center location

$\frac{a}{b}$	h	$\frac{c}{a}$	θ	$\frac{x_c}{b}$
2	0.03	0.5	90	0.5

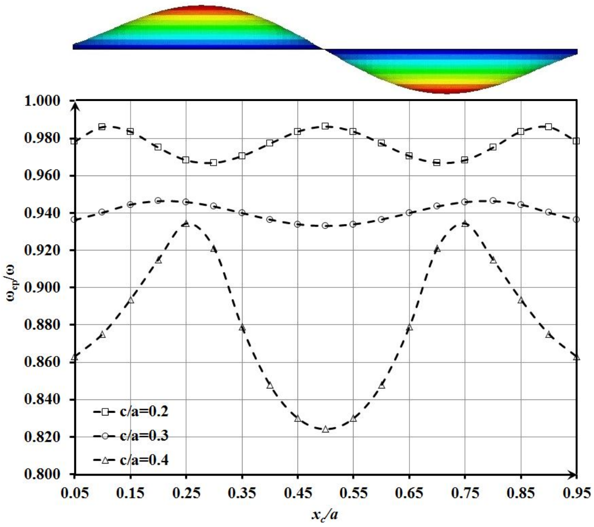


(a) 2nd mode

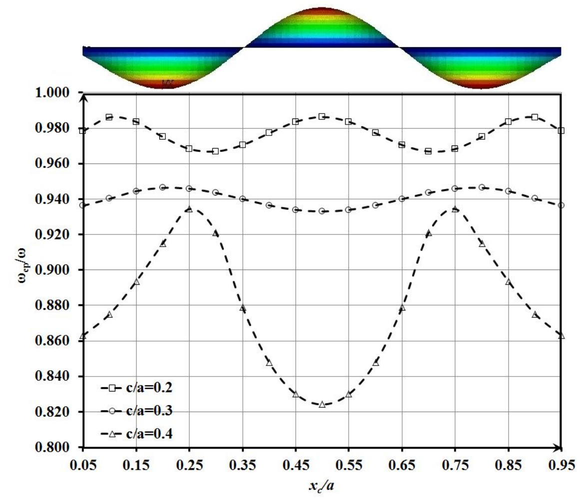


(b) 3rd mode

Figure 23. Effect of location of crack centre on the natural frequency ratio of the simply-supported plate



(a) 2nd mode



(b) 3rd mode

Figure 24. Effect of location of crack centre on the natural frequency ratio of the clamped plate

4. Conclusions

In the current paper, modal analysis of cracked plate by the aid of a thorough finite element model was made possible. Numerical results of this model are much more precise than those of previous studies. The variations of all parameters affecting the free vibration of cracked plate and different boundary conditions can be analyzed by this model. The studies in this paper show that mechanical properties of the materials and plate thickness do not affect the ratio of natural frequencies of a cracked plate to those of an intact plate and it can be concluded that the extent of the influence of these parameters on natural frequencies of a cracked plate is similar to that of an intact plate.
For example, thickness has a linear relationship with the natural frequencies of an intact plate and a cracked plate. The following examinations reveal that modal shapes influence stress intensity factors on crack tip and then, the stiffness of the plate by influencing the forces working on crack planes and thus, they affect relative frequencies. In fact, with the increase (or decrease) in stress intensity factors, relative frequencies decrease (or increase); that is, the effect of crack increases (or decrease). For instance, when the crack is perpendicular to nodal line, modal shapes change less compared to when it is parallel to nodal line and then, it would have less influence on stress intensity factors and natural frequencies.

List of Symbols

a	Plate length (m)
b	Plate breadth (m)
c	Crack length (m)
E	Young's modulus (N/m^2)
h	Plate thickness (m)
U_a	Elastic strain energy released by crack (J)
U_E	Elastic strain energy of cracked plate (J)
U_i	Elastic strain energy of intact plate (J)
x_c	Longitudinal coordinate of the crack (m)
y_c	Transverse coordinate of the crack (m)
ν	Poisson's ratio
θ	Crack angle ($degree$)
μ	Mass per unit area (kg/m^2)
ρ	Density (kg/m^3)
ν	Poisson's ratio
ω	Natural frequency of intact plate ($1/s$)
ω_{cp}	Natural frequency of cracked plate ($1/s$)

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