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Dynamics of a Single Point Mooring Marine Aquaculture Cage as a Simple Vibrating System

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ABSTRACT

The increasing world demand for fish cannot be met by capture fisheries. Aquaculture production is increasing and nowadays cage culture has an important role in meeting the world's fish demand. The design of the physical structure of a cage is determined by the oceanographic conditions of the culture site. Each design is site-specific and knowledge of the topography, wind force and direction, prevalence of storms or monsoons, wave loads, current velocity and water depths are important parameters for consideration. Because of all these reasons the design of an aquaculture cage system is very complex and difficult task. Hence, it is essential to select a proper site, ideal construction materials, proper designing, suitable mooring, good management etc. in bringing out a cage culture production more profitable and economical. A six degree of freedom model is considered to find out the motions and forces acting on the cage. The tensions in the mooring chain and the net twine tension were predicted based on numerical simulation.

1. Introduction

Mariculture is cultivation of marine organisms in the open sea by enclosing a part of it using a cage structure. It is now one of the most promising methods of fish production all over the world. It is reported to account for 36% of the total aquaculture production of 59.4 million tonnes per year. Total aquaculture production of India is 2.2 million tonnes per year while production through mariculture is insignificant. However, India has a lot of potentiality to develop mariculture; as it has long coast line of about 8129 km, continental shelf of 0.5 million square km and Exclusive Economic Zone (EEZ) of 2.2 million square km. The need to study cage dynamics arises because of the various technical failures taking place and also some accidents that have taken place which makes it important to predict cage motions.

In the paper by Huang et. al [1] the dynamic analysis of net cages was done based on the "lumped mass method and net plane element" concepts. The advantages of this concept are the whole net-cage system can be decomposed into flexible net plane elements, line elements, bottom weights, and rigid solid body, such as floating collar which may be divided into several straight tube elements. Zhao et al. [2] developed a numerical model to simulate the tension distribution in the fishing cage in current. The fishing net can be modeled as a series of lumped point masses that are interconnected with springs without mass.

A numerical model for analyzing dynamic properties of a net-cage system exposed in the open sea is proposed. The forces on the cage system are computed from the above references. In the present study, we have presented a six degree of freedom model for a single point mooring cage and tried to study the various motions of the cage. Both first order linear shallow water dispersion relation and fifth order Stokes wave theory are used in the model for determining wave and damping forces. A multidirectional wave field has been considered to simulate real open ocean conditions.

We have also considered that the floating collar and the sinker move independently as the net is a flexible system. In the paper by Cha et. al [3], the dynamic response simulation of heavy cargo suspended by a floating crane is performed. The dynamic equations of the motions of the floating crane and the heavy cargo are considered by the coupled equations. This resembles our floating collar system having a suspended sinker weight.

2. Cage Body Description

The common components of any type of cage system are: floating system, mooring system, anchor system, net cage and services system as shown in Figure 1.

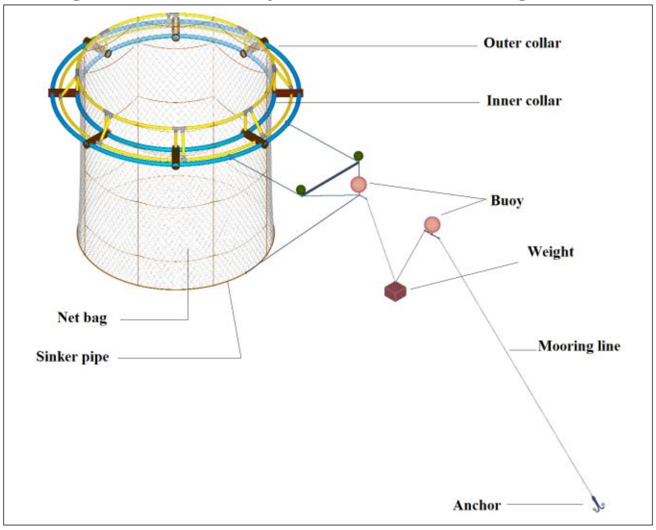

Figure 1. Components of a cage structure

Floating system: Provides buoyancy and holds the system at a suitable level in the surface of the water. In some cages this component is an important part to hold the shape of the cage. Common flotation materials include metal or plastic drums, high density polyethylene (HDPE) pipes, rubber tires and metal drums coated with tar or fiber glass. The buoyant force varies depending on size and materials used.

Services system: This is the system required for providing operating and maintenance services, for example: feeding, cleaning, monitoring or grading. One way to provide this is by a catwalk around the cage or along part of the cage. Some cages use their flotation collars like catwalks and access for these services. These flotation collars are made of metal or plastic pipes with or without additional internal or external floats. The assembly with the cage or its structure is by connectors or ties using ropes. The size depends on the cage design. The initial cost of catwalks could be relatively high, but the services are indispensable.

Cage bag: The function of the bag is to contain and protect the fish and to provide a marine habitat. The net is normally flexible and made of synthetic netting of nylon or polythene fibres reinforced with polythene ropes. The nets are kept stretched vertically with weights at the bottom of the cage or fastened by rope to the framework depending of the type of cages (Chua and Tech) [4]. Rigid cages made of metal netting (galvanized mesh, copper-nickel mesh or vinyl-coated mesh) mounted on rigid metal frameworks also are used. The flexible net bag is most used due to cost (Huguenin) [5].

Mooring system: This holds the cage in the suitable position according to the direction and depth decided in the design, and sometimes helps to maintain the shape of the cage. The mooring joins the cage to the anchor system. A mooring system must be powerful enough to resist the worst possible combination of the forces of currents, wind and waves without moving or breaking up. The materials used in the mooring systems are sea steel lines, chains, reinforced plastic ropes and mechanical connectors. The mooring force capacity depends on both the material and size, and can be adjusted to the requirements. Attachment to the system is by metallic connectors and ties.

Anchor system: This holds the cage and all the components in a particular site in the seabed and is connected to the cage by the mooring system. There are basically three types: pile anchors, dead weight anchors and anchors that get their strength by engaging with the seabed. Pile anchors are buried piles in the seabed, they are effective, especially for systems where a small space is necessary, they are driven into the seabed usually by a pile hammer from a barge on the surface; but, they are expensive to buy and install. Dead weight anchors are usually concrete blocks. Their one big advantage is that they are fairly consistent in holding power. The third type is mooring anchors which have to hold into a particular seabed when pulled from one direction only; they are made of steel and should slip easily into the seabed without disturbing the soil in front of it. The anchors are joined to the mooring system usually by chains and metallic connectors.

3. Mathematical Formulation

The coordinate system of the cage is shown in Figure 2. We have considered a six degree of freedom system comprising of surge, heave, sway, roll, pitch and yaw. The surge velocity is u, heave velocity is w, sway velocity is v, roll angular velocity is p, pitch angular velocity is q and yaw angular velocity is r. The coordinates of collar knot is x_k,y_k, z_k and center of gravity is x_g, y_g, z_g, the center of floating collar. The coordinates of the center of gravity are x_g = 0, y_g = 0, z_g = 0. We formulate the model using the above details to get equations of motion given by Eq. (1).

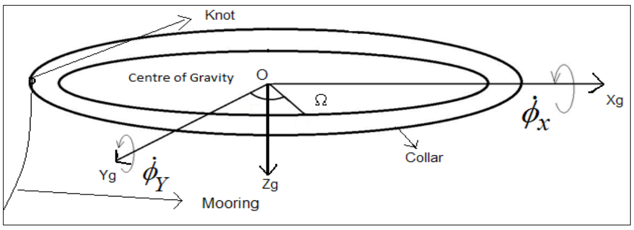

Figure 2. Coordinate system of the mooring cage

$$m[\dot{u}+qw-rv+(\dot{q}+rp)z_g-(q^2+r^2)x_g]=$$
$$X_h-X_D-X_{mooring}$$
$$m[\dot{v}+ru-pw+(pq+\dot{r})x_g-(rq-\dot{p})z_g]=$$
$$Y_h-Y_{mooring}-Y_D$$
$$m[\dot{w}+pv-qu+(pr-\dot{q})x_g-(p^2+q^2)z_g]=$$
$$Z_v-Z_D-W+B-Z_{mooring}$$
$$I_x\dot{p}+I_{xz}r+(I_z-I_y)rq+I_{xz}pq-m[z_g(\dot{v}+ru-pw)]=$$
$$K_{Force}+K_{Restoring}+K_{mooring}$$
$$I_y\dot{q}+(I_x-I_z)pr+I_{xz}(r^2-p^2)+m[z_g(\dot{u}+qw-ru)-x_g(\dot{w}+pv-qu)]=$$
$$M_{Force}+M_{Restoring}+M_{mooring}$$
$$I_z\dot{r}+(I_y-I_x)pq+I_{xz}p-I_{xz}qr+m[x_g(\dot{v}+ru-pw)]=$$
$$N_{Force}+N_{Restoring}+N_{mooring} \tag{1}$$

On the right hand side of Eq. (1), subscript 'h' indicates horizontal force, subscript 'v' denotes vertical direction, subscript'd' denotes drag force and subscript 'mooring' denotes forces due to mooring tension.

3.1 External Wave and current force

The modified Morison equation (Brebbia and Walker) [6] to calculate the wave and current forces acting on the cage structure is shown in Eq.(2).

∇ is the displaced volume of structural member

Relative velocity, V_R is defined by Eq. (3). V is the water particle velocity and R is the cage velocity. The cage velocity can be substituted by u, v and w for surge, sway and heave respectively.

$$X_h=\frac{1}{2}\rho C_D A V_R\left|V_R\right|+\rho\nabla K_M\frac{\partial V_R}{\partial t}+\rho\nabla\frac{\partial V_w}{\partial t} \tag{2}$$

$$V_R=V_w-\dot{R} \tag{3}$$

The first term on the right hand side of Eq. (2) is regarded as the drag force, second term describes the added mass and the third term is called the inertia force. Added mass is dependent on the structure shape and fluid acceleration relative to the structure. The hydrodynamic force can be written as shown in Eq. (4)

$$X_h=\frac{1}{2}\rho C_D A V_R\left|V_R\right|+\rho\nabla C_M\frac{\partial V_w}{\partial t}-\rho\nabla K_M\frac{\partial \dot{R}}{\partial t} \tag{4}$$

where $C_M = 1 + K_M$ is the inertia coefficient.

We assumed a uniform current flowing along the x direction and a horizontal seabed, therefore the velocity potential is given by Eq. (5) according to linear theory.

$$\phi=-(U_x x)+\frac{Hg}{2\omega}\frac{\cosh(k(h+z))}{\cosh(kh)}\cos(kx-\omega t) \tag{5}$$

The velocity fields given by Eqs. (6) and (7) and the accelerations given by Eqs. (8) and (9) are applied to Morison equation to compute hydrodynamic forces on net cage system.

$$u_w=U+\frac{Hgk}{2\omega}\frac{\cosh(k(h+z))}{\cosh(kh)}\sin(kx-\omega t) \tag{6}$$

$$w_w=-\frac{Hgk}{2\omega}\frac{\sinh(k(h+z))}{\cosh(kh)}\cos(kx-\omega t) \tag{7}$$

$$\frac{\partial u_w}{\partial t}=-\frac{Hgk}{2}\frac{\cosh(k(h+z))}{\cosh(kh)}\cos(kx-\omega t) \tag{8}$$

$$\frac{\partial w_w}{\partial t}=-\frac{Hgk}{2}\frac{\sinh(k(h+z))}{\cosh(kh)}\sin(kx-\omega t) \tag{9}$$

The wave elevation and velocity potential for Stokes wave are given by Stokes theory. A unidirectional current in x direction having a velocity of 0.5 m/s has been considered in addition to the Stokes wave for force computation. A wave field at an angle β with respect to the x direction has been considered in addition to a unidirectional current with a velocity of 0.5 m/s in x direction. The potential used for calculating water particle velocities in this case is shown in Eq. (10). In this case the roll and the sway motion of the cage also become important and subsequently have been calculated using the coupled equations of motion described earlier.

$$\phi=Ux+\frac{ag}{\omega}\frac{\cosh(k(z+d))}{\cosh(kd)}\sin(kx\cos\beta+ky\sin\beta-\omega t) \tag{10}$$

For force calculation on the net, the net has been discretized into individual mesh elements along the circumference and the depth. The net force is then integrated over the circumference and depth. The inertia coefficient of net elements in the present study is considered as $C_{mn} = 1.2$, $C_{mt} = 1$, where C_{mn} and C_{mt} are the normal and tangential inertia coefficients of mesh bar. Assuming a water particle at the center of net element moving at relative velocity V_R, the drag and lift force are given as shown by Eqs.(11) and (12).

$$F_D=\frac{1}{2}\rho C_{Dn}A_{net}\left|V_R\right|^2\hat{e}_n \tag{11}$$

$$F_{Dt}=\frac{1}{2}\rho C_{Dt}A_{net}\left|V_R\right|^2\hat{e}_t \tag{12}$$

The evaluation of the drag coefficient for the net, C_D, is a matter of discussion in literature, and a complicated matter. Lader et al. [7] and Fredriksson [8] suggest drag and lift coefficients (C_D and C_L) for modeling of a simple net exposed to waves and currents in three dimensions. The drag and lift coefficients selected in our formulation are given by Eqs. (13) and (14).

$$C_D = 0.04 + (-0.04 + S_n - 1.24S_n^2 + 13.7S_n^3)\cos(\alpha) \quad (13)$$

$$C_L = (0.57S_n - 3.54S_n^2 + 10.1S_n^3)\sin(2\alpha) \quad (14)$$

where S_n is the solidity of the net. Solidity of net is defined as the ratio between the projected area of the net and the total area enclosed by the net.

The details of the external force calculation on the collar can be found in Huang et al. [1,9]. To calculate the inertia force and drag force on the collar we have discretized the collar into small elements along the circumference as shown in Figure 3. The effect of these small elements on the flow field is negligible. The drag forces on a circular cylinder can be decomposed into two parts, one is normal to the pipe and the other is tangential to the pipe. Considering the normal and tangential coefficients we have calculated the horizontal and vertical drag and inertia force on each such element and integrated over the entire collar. The drag force on the collar elements can be calculated as shown in Eqs. (15) and (16).

$$F_{Dn} = \frac{1}{2}\rho C_{Dn} A_n |V_R|^2 \hat{e}_n \quad (15)$$

$$F_{Dt} = \frac{1}{2}\rho C_{Dt} A_t |V_R|^2 \hat{e}_t \quad (16)$$

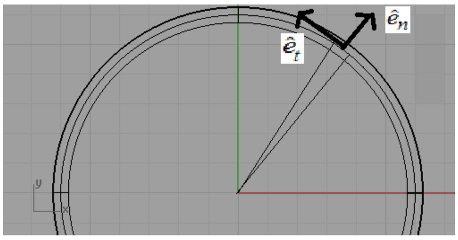

Figure 3. Individual collar element normal and tangent

The areas $A_n = hl$, $A_t = pl$ are as shown in Figure 4, l is the length of individual segment. The normal drag coefficient C_{Dn} for the collar elements is taken as 0.6 and the tangential drag coefficient C_{Dt} is taken to be 0.1 as suggested by Gudmestad et al. [10]. The inertia coefficient C_M is taken to be be 1.3.

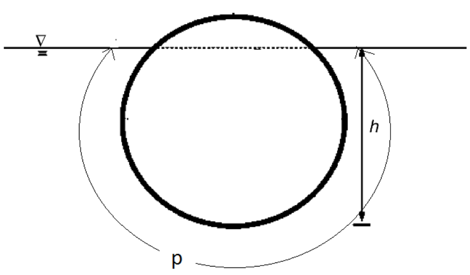

Figure 4. Cross section showing parameters for normal and transverse area

We have taken a 12 mm D-Link mild steel chain. Using Young's modulus of steel (200 MPa), cross sectional area of chain and length, we can calculate stiffness of mooring using Eq. (17).

$$K = \frac{P}{\delta L} = \frac{E}{AL} \quad (17)$$

Therefore the mooring force is given as

$$F_M = K\varepsilon, \text{ if } \varepsilon >= 0$$

$$F_M = 0, \quad \text{if } \varepsilon < 0$$

Where $\varepsilon = \dfrac{l_{extended} - l_{initial}}{l_{initial}}$

3.2 Buoyancy and Weight

The floating collars provide buoyancy to the cage structure. It is modeled as a rigid tube. There are three floatation conditions (i) centerline of the collar is above the water level (ii) centerline of the collar is below water level (iii) collar is completely submerged. These cases are shown in Figure 5. The corresponding mathematical formulation is given by Eqs. (18 ~ 20).

$$A = \frac{\theta}{2\pi}\pi R^2 - 2(R-H)\sqrt{R^2 - (R-H)^2} \quad (18)$$

Where $\cos(\theta/2) = \dfrac{(R-H)}{R}$

$$A = \pi R^2 - \left(\frac{\theta}{2\pi}\pi R^2 - 2(R-H')\sqrt{R^2 - (R-H')^2}\right) \quad (19)$$

Where $\cos(\theta/2) = \dfrac{(R-H')}{R}$ And $H' = 2R - H$

$$A = \pi R^2_{collar} \quad (20)$$

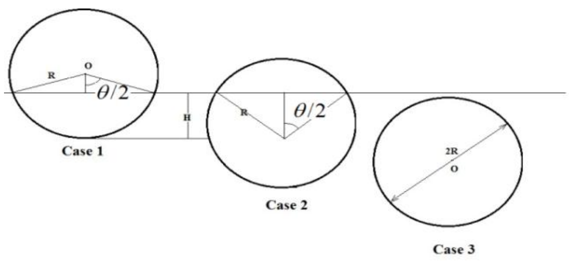

Figure 5. Cross section in different conditions of submergence

The buoyancy force in Eq. (1) is given by Eq. (21). Net twine shape is shown in Figure 6. The immersed volume of a single net twine is given by Eq. (22). The total buoyancy of the net is given by Eq. (23) ,

$$B_{collar} = \rho P_c g A \quad (21)$$

$$V_{mesh} = \frac{\pi}{4} D_{twine}^{2} S_{mesh} \tag{22}$$

$$B_{net} = \rho g \sum_{1}^{m} \sum_{1}^{n} V_{mesh} \tag{23}$$

$$n = \{\frac{P_{Cage}}{S_{mesh}}\}_{outernet} + \{\frac{P_{Cage}}{S_{mesh}}\}_{innernet}$$

$$m = \{\frac{D_{Cage}}{S_{mesh}}\}_{outernet} + \{\frac{D_{Cage}}{S_{mesh}}\}_{innernet}$$

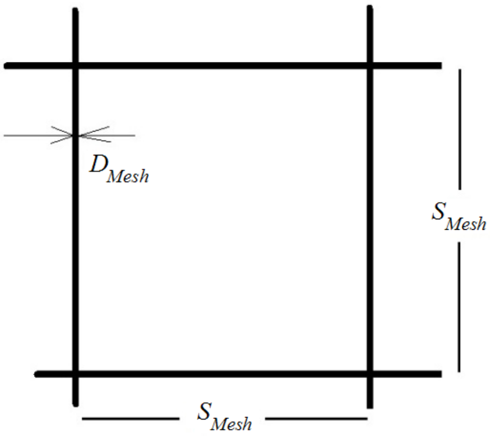

Figure 6. Square Net mesh element

Sinker pipe is shown in Figure 7. Volume of sinker (Vs) pipe is given by Eq. (24). Buoyancy provided by sinker pipe is given by Eq. (25)

$$V_s = \frac{\pi}{4} D_s^2 P_s \tag{24}$$

$$B_S = V_S \rho g \tag{25}$$

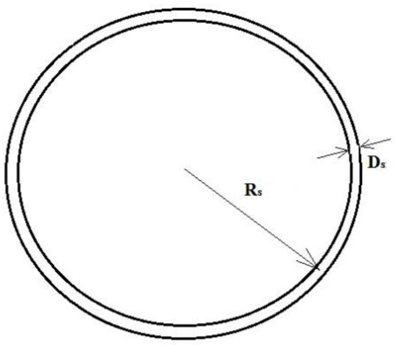

Figure 7. Plan view of sinker pipe

3.3 Restoring moment of collar in pitch or roll

The inclined plane in Figure 8 shows the condition when the collar pitches by an angle (α) and shows the water surface intersecting the collar at different locations. As shown in Figure 9, at different x locations where x = D/2 cos(θ) (0 <= θ <= 2π for the tube) the line cuts the individual tube elements at different heights h, assuming that the inclined line is horizontal because angle of inclination in pitch is small. ψ is the same as θ in the buoyancy section of floating collar and accordingly the increased or decreased area can be calculated from Eqs. (18) and (19) in the buoyancy section using values of h and θ from Eqs. (26) and (27).

$$h = x\alpha \tag{26}$$

$$\psi = \sin^{-1}(\frac{h}{D/2}) \tag{27}$$

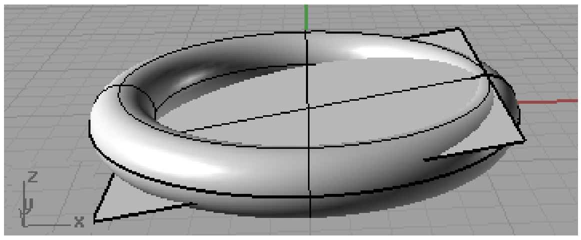

Figure 8. Inclined water surface intersecting collar

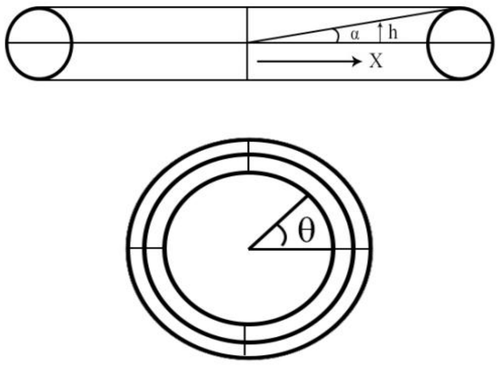

Figure 9. Front and top view of Collar

The restoring moment per unit length of an individual section is given by Eq. (28). In Case the collar is out of water on one side and submerged on the other side the values are given by Eqs. (29) and (30).

$$K_{restoting} = \rho A_{in/dc} g x \tag{28}$$

$$\theta_{fix} = a\tan(\frac{d}{D}) \tag{29}$$

$$H = x\theta_{fix} \tag{30}$$

3.4 Modeling the net and sinker pipe as a Hanging Mass System

When we consider the motions of the cage we have to take into account the fact that the collar and net (plus sinker) move independently of each other. Therefore, to make the model more realistic we have considered collar (rigid tube) as one system having a certain mass and the sinker and the net as a lumped mass attached to the collar by means of a spring whose stiffness is given by Eq. (17). The model is shown in Figure 10. The wave force and drag force on the collar are taken to act on the collar system and the wave and drag force on the hanging mass are found by summing up the forces on the net and the sinker.

The body coordinates and the earth coordinates are mentioned below:

- Position of collar knot: $r_k = [x_k, y_k, z_k]$
- Position of collar centre of gravity: $r_c = [x_c, y_c, z_c]$
- Position of hanging mass: $r_n = [x_n, y_n, z_n]$
- Angular rotations of collar: $\theta_c = [\theta_c, \varphi_c, \psi_c]$
- Angular rotation of hanging mass: $\theta_b = [\theta_n, \varphi_n, \psi_n]$

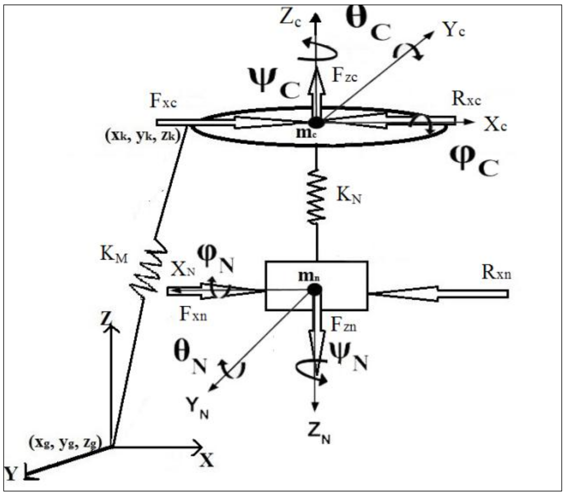

Figure 10. Modeling the cage as a coupled hanging mass system

As we are writing equations of motion in body frame of reference we need to transform the coordinates from body frame to Earth frame for equations to be compatible. Also, as the water particle velocities are in the Earth frame and the forces and moments acting on the cage are in the body frame, therefore we need to apply an inverse transformation to transform the forces and moments from earth frame to body frame. We use the method of Euler angle transformation for change of coordinates and the order of transformation from body frame to earth frame is roll - pitch - yaw and inverse in case of earth to body frame. Eqs. (31 ~ 33) show the transformation matrix used for yaw pitch and roll respectively.

$$T_z = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{31}$$

$$T_y = \begin{bmatrix} \cos\varphi & 0 & \sin\varphi \\ 0 & 1 & 0 \\ -\sin\varphi & 0 & \cos\varphi \end{bmatrix} \tag{32}$$

$$T_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \tag{33}$$

The transformation matrix for wave particle velocities from earth coordinates to body coordinates is given by Eq. (34) where θ_c, φ_c, ψ_c are Euler angles. The transformation matrix for hanging mass to collar centre of gravity coordinates is given by Eq. (35) where θ_n, φ_n, ψ_n are the Euler angles. After solving the motion equations in the body frame we can transfer the velocities and positions to earth coordinates using the Eq. (35) where θ_c, φ_c, ψ_c are the Euler angles.

$$R = \left[T_z T_y T_x\right]^{-1} \tag{34}$$

$$R_1 = T_x T_y T_z \tag{35}$$

For finding out the drag force on the net and the sinker we have used the net velocity of the collar and the hanging mass as shown in Eq. (36). This velocity is then input in Eq. (4) for finding out damping force in x, y and z direction. The force computation for the collar and hanging mass is given by Eq. (37) and the equations of motion are given by Eqs. (38 ~ 40). We have considered motion in only X, Y and Z directions as the spring cannot sustain any moment so there is no pitch and roll motion.

$$u_{total} = u_{hang} + u_{collar} \tag{36}$$

$$F_{Hang} = F_{Net} + F_{Sinker} \tag{37}$$

$$m_{hang}\dot{u}_{hang} = F_{xhang} - k_{hang}\Delta x_{hang} \tag{38}$$

$$m_{hang}\dot{v}_{hang} = F_{yhang} - k_{hang}\Delta y_{hang} \tag{39}$$

$$m_{hang}\dot{w}_{hang} = F_{yhang} - k_{hang}\Delta z_{hang} - mg \tag{40}$$

k_{hang} is stiffness of net , Δz_{hang} is extension in z direction, Δy_{hang} is extension in y direction, Δx_{hang} is extension in x direction and m is mass of the sinker. The net stiffness is considered to be linear and calculated in the same way as mooring stiffness from Eq. (29). The area of cross section is taken to be the sum of all the net twines in the vertical direction and length is taken as the depth of net. As there are two nets the total stiffness is the sum of the outer and inner net.

4. Results and Discussion

The parameters used for calculations are given in Table 1.

Table 1. Simulation Parameters

Characteristic	Value
Total Cage Mass	750 kg
Collar pipe diameter (inner and outer)	0.26 m
Outer Collar diameter	6 m
Inner Collar diameter	5 m
Sinker pipe diameter	8 m
Depth of Cage	6 m
Net mesh size (outer)	80 mm
Net mesh size (inner)	35 mm
Net twine Diameter	2.5 mm
Net Stiffness	866240 N/m
Net material density	950 kg/m^3
Twine ultimate tensile force	49 N
Solidity ratio of net	0.0625
Mooring Stiffness	1023121 N/m
Mooring length	37.94 m
Mooring ultimate tensile force	56 kN

We have first considered uncoupled single degree of freedom in heave to check for the validity and correctness of the model. The following result is shown for wave height H = 2.5 m and Wave period T = 8 s with current = 0.5 m/s. Figure 11 shows the heave in single degree of freedom. The wavelength for 8 second wave is 75.8 m and the diameter of the cage is 6 m, therefore we can assume that the collar moves up and down with the wave. The heave motion shown in Figure 8 is in addition to this motion. Therefore, the heave motion is shown relative to instantaneous free surface in all cases. It shows that two motions are superimposed, one is that of the wave with a period of 8 seconds and other is due to mooring stiffness with period of approximately 0.5 seconds.

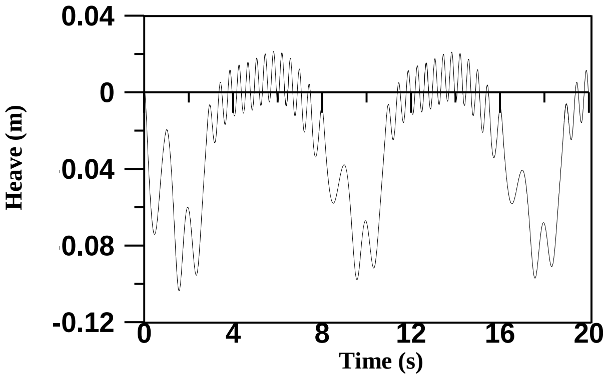

Figure 11. Heave for single degree of freedom

4.1 3 Degree of freedom model

Further, we have taken a three degree of freedom system including heave, surge and pitch. We have considered that the collar, net and sinker move together as a rigid body. The following results are shown for wave height H = 2.5 m and Wave period T = 8 s with current = 0.5 m/s. Figure 12 shows heave motion, surge motion and pitch motion. It is seen that due to non-linearity of model the oscillations are significantly different than single degree of freedom. Pitch motion has a higher positive value because on one side there is a higher stiffness due to the mooring which restricts the motion. Figure 13 shows the mooring tension. It is seen that average mooring tension is around 3 kN for the time during which it is under tension. The mooring remains unstretched for a considerable amount of time during which tension is zero.

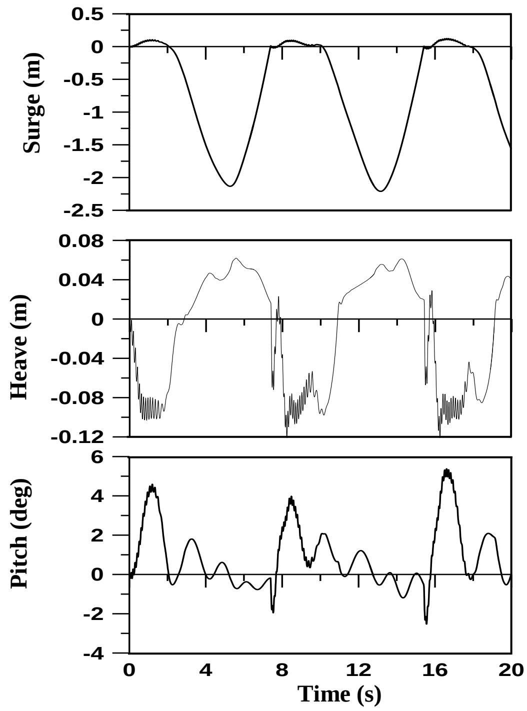

Figure 12. Displacements for 3 degree of freedom model in Earth coordinate

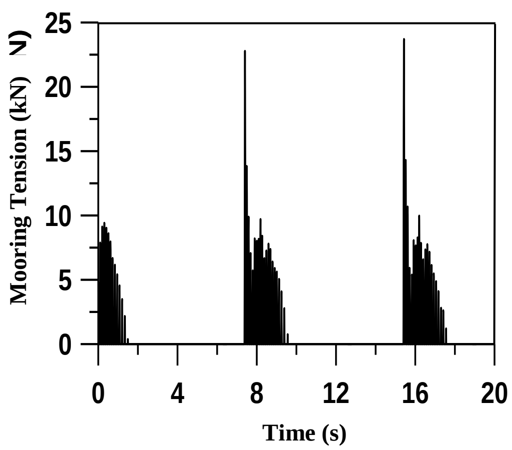

Figure 13. Tension in mooring for 3 degree of freedom model

4.2 5 Degree of freedom model including hanging Mass and oblique seas

The following results are shown for wave height H = 2.5 m and Wave period T = 8 s with current = 0.5 m/s and sea at an angle of 45° from X axis. Figure 14 shows heave motion, surge motion and sway motion of collar. Figure 15 shows pitch motion and roll motion of collar.

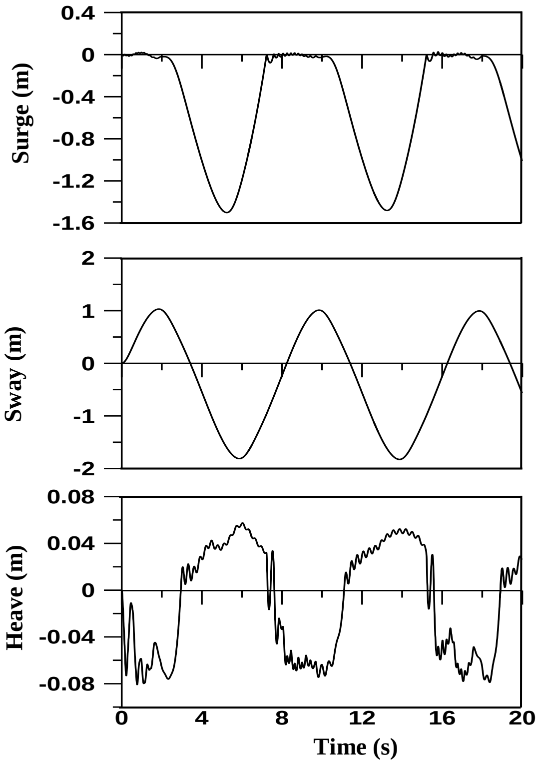

Figure 14. Displacements of collar in Earth coordinate for 5 degree of freedom model

Figure 16 shows heave motion of the hanging mass, surge motion of the hanging mass and sway motion of the hanging mass. It is seen there is a reduction in the magnitude of motions of the collar because of increase in degrees of freedom. It is also seen that roll motion of the collar is symmetric because there is no additional mooring stiffness in roll direction. We can also see there are significant motions of the sinker and therefore we cannot ignore the motion of the sinker in addition to the motion of the collar. As the net is connected to the collar by means of ropes the connection is not rigid. Also the net is deformable and has been considered as a linear spring connected between the floating ring and the sinker, therefore the sinker can move in three directions relative to the collar. It is seen that there is a difference in the heave motions of the collar and the sinker because the sinker has no buoyancy term and is affected only by sinusoidal wave force and damping force.

Figure 17 shows tension per individual twine and mooring line tension for a 5 degree of freedom hanging mass model. It can be seen that unlike the mooring the twines are always in tension with an average value of tension being 6 N per twine which is below the ultimate tensile limit as shown in Table 1.

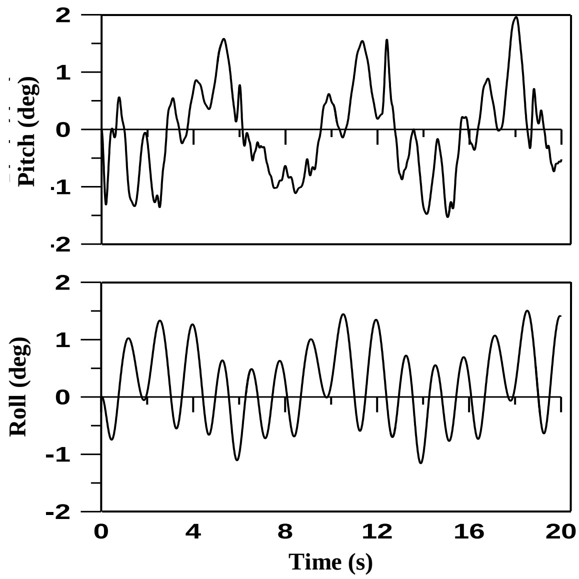

Figure 15. Angular displacements of collar in Earth coordinate for 5 degree of freedom model

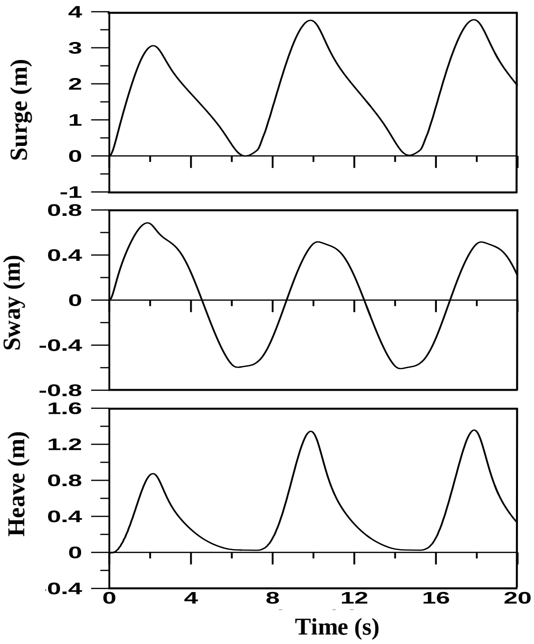

Figure 16. Displacements of net and sinker in earth coordinate (hanging mass) for 5 degree of freedom model

There is a reduction in the maximum value of tension for mooring as compared to 3 degree of freedom model but the average tension are of comparable values.

4.3 Stokes 5th order wave using 3 Degrees of freedom

The following results are shown for wave height H = 2.5 m and Wave period T = 8 s with current = 0.5 m/s and sea at an angle of 45° from X axis. Figure 18 shows the mooring line tension for Stokes 5th order wave theory using 3 degree of freedom model.

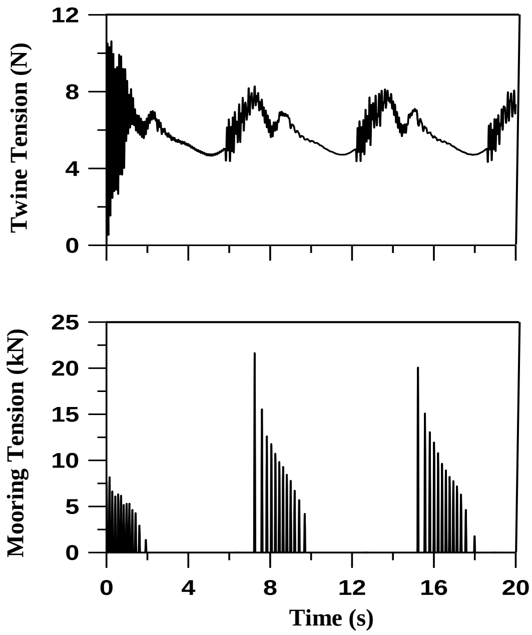

Figure 17. Tension in twine and mooring for 5 degree of freedom model

It can be seen that the tension is zero most of the duration as compared to other two cases. The average tension calculated is around 8.5 kN for the time during which it is under tension. There is a sudden variation in tension because when the mooring gets stretched, due to high stiffness the collar tends to return to original position rapidly. Figure 19 shows heave motion, surge motion and pitch motion for a Stokes 5th order wave. We have considered the 3 degree of freedom, rigid cylinder model for applying Stokes 5th order wave theory. There is an increase in the magnitude of wave forces for stokes 5th order wave, though we do not see a corresponding increase in magnitude of motions because motions depends not only on magnitude of forces but also the wave slope.

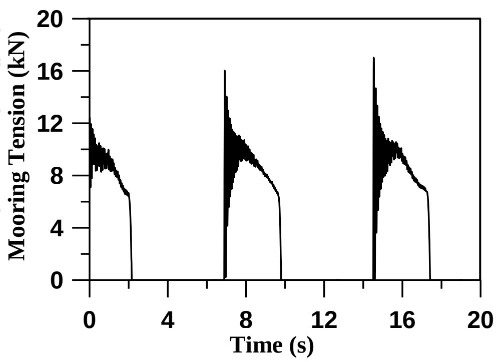

Figure 18. Tension in mooring for Stokes 5th order wave

4.4 Stokes 5th order wave using 5 Degrees of freedom

The following results are shown for wave height H = 2.5 m and Wave period T = 8 s with current = 0.5 m/s and sea at an angle of 45° from X axis. Figure 20 shows the heave, surge and sway motion for this case. It is seen that in this case surge never crosses the initial position but mooring gets stretched mainly due to contribution from heave.

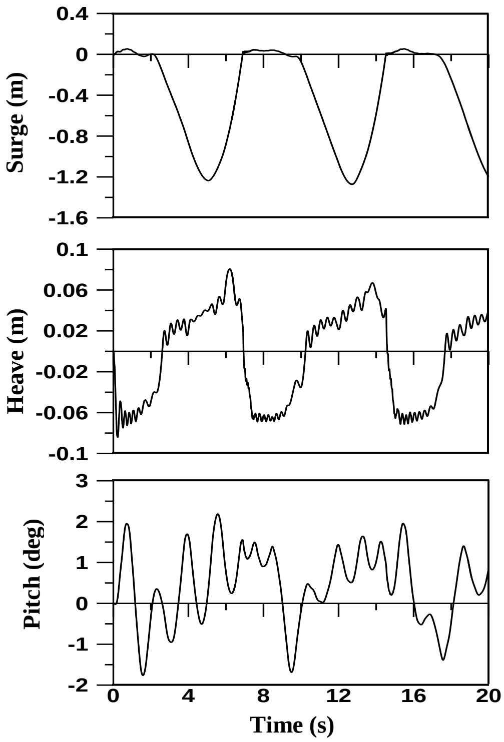

Figure 19. Displacements for 5th order Stokes wave in Earth coordinate for 3 degree of freedom model

Figure 22 shows mooring line tension and individual twine for Stokes 5th order wave theory using 5 degree of freedom model. The average mooring tension calculated is around 5.5 kN for the time during which mooring is under tension. The average twine tension is around 6 N. We can see reduction in value in this case as compared to earlier cases. Figure 23 shows heave motion, surge motion and sway motion for the hanging mass i.e. sinker. We can see there is a reduction in magnitude of motions as compared to 5 degree of freedom model with linear wave. There is an increase in the magnitude of wave forces for stokes 5th order wave, though we do not see a corresponding increase in magnitude of motions because motions depends not only on magnitude of forces but also the wave slope. We have compared our results with James [12], who has presented experimental results on the same mariculture cage. We see that the twine tension is in very close agreement with the experimental values but our model predicts a higher value of mooring tension than that observed by experiments.

4.5 Effect of various parameters on tension and motions

We have studied the effect of different parameters like wave period, water depth and diameter of cage on the motions and forces acting on the cage. By analyzing this data we can decide the cage location, size and mooring line characteristics. This will help in robust and efficient design.

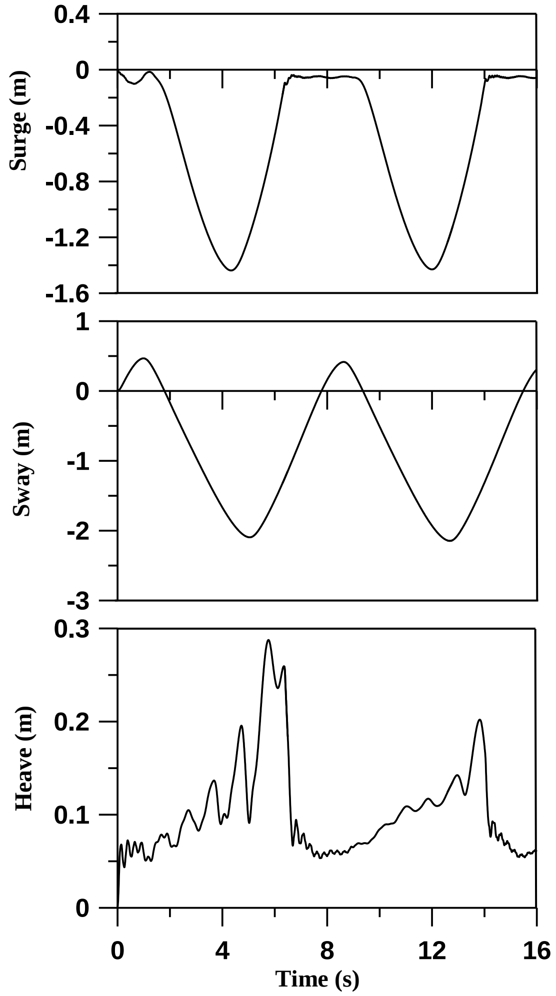

Figure 20. Displacements of collar in Earth coordinate for 5 degree of freedom model for Stokes 5th order wave

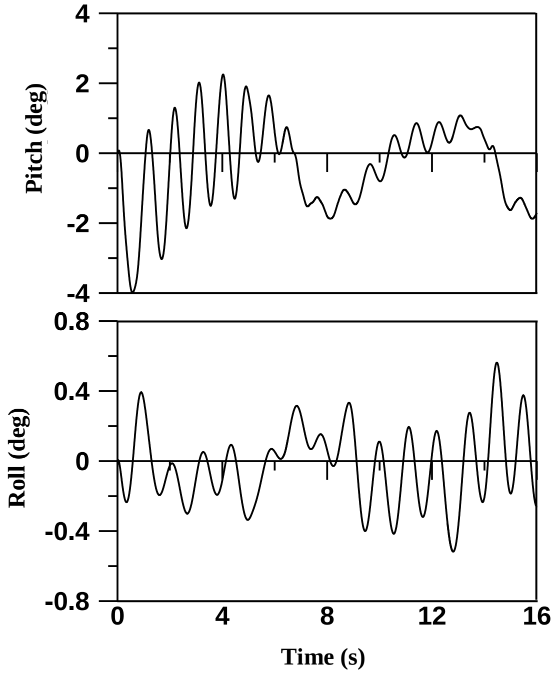

Figure 21. Angular displacements of collar in Earth coordinate for 5 degree of freedom model with Stokes 5th order wave

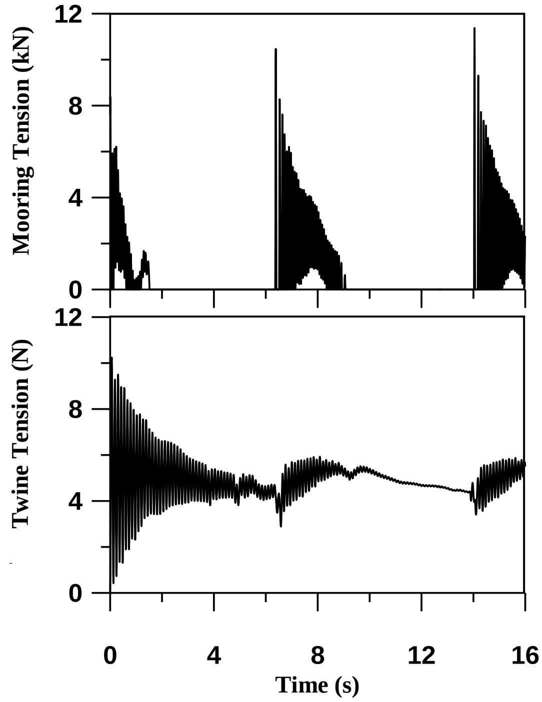

Figure 22. Tension in twine and mooring for 5 degree of freedom model with Stokes 5th order wave

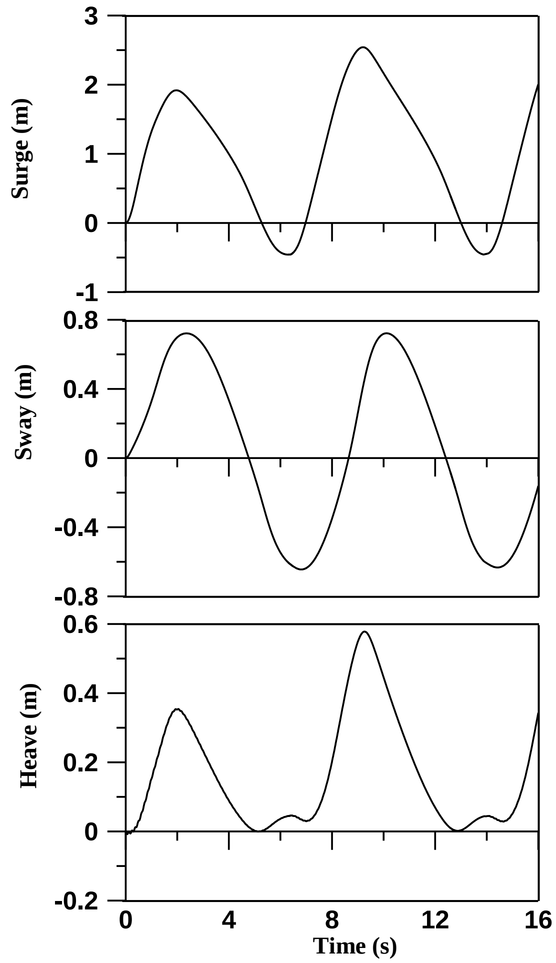

Figure 23. Displacements of net and sinker in earth coordinate (hanging mass) for 5 degree of freedom model with Stokes 5th order wave

4.5.1 Influence of time period of waves

We have studied the effect of changing the time period of exciting waves from a range of 8 seconds to 12 seconds, wave height H = 2 m , current = 0.5 m/s and sea at an angle of 45° from X axis.

We can see that the heave motion and surge motion decrease as time period increases as shown in Figure 24. The sway motion increases as time period increases. Also we can see in Figure 25 that the pitch and roll motion decreases with increase in time period. It can be inferred that time period has a linear effect on surge, heave and pitch. Figure 26 shows that the mooring tension decreases with increase in time period and this is in validation with Huang et. al [13]. It can be seen that there is a significant decrease in tension from 11.5 kN to 9 kN.

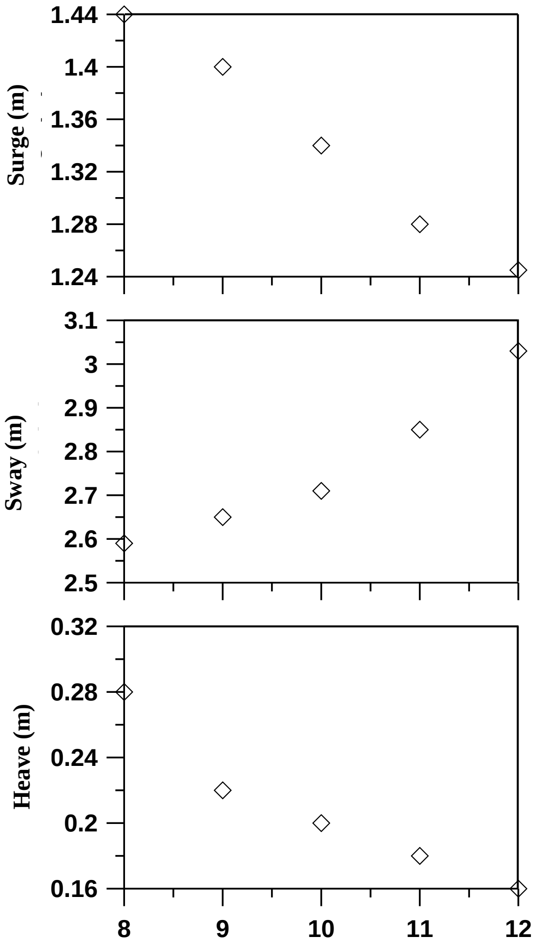

Figure 24. Response of cage against time period of wave for the 5 degree of freedom Stokes wave model

4.5.2 Influence of depth and cage diameter

Results have been shown for cage diameters in order of 6 m, 12 m ,15 m and water depth of 12 m and 20 m with wave height H = 2m , Time period T = 8 s, current = 0.5 m/s and sea at an angle of 45° from X axis.

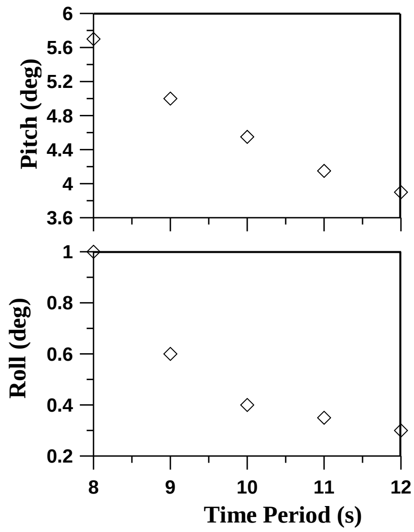

Figure 25. Angular Response of cage against time period of wave for the 5 degree of freedom Stokes wave model

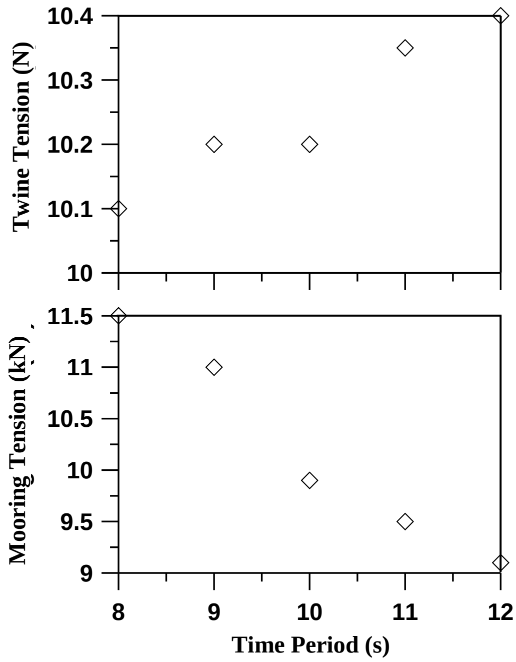

Figure 26. Mooring and twine tension against time period of wave for the 5 degree of freedom Stokes wave model

Figure 27 shows that the surge and sway decrease as the cage diameter increases. This is because large cage has larger mass which increases the inertia. The drag force increase but less compared to inertial force. Therefore, there is a decrease in motions. It also shows that at higher depth the magnitude of motions is reduced. This is because the wave force due to Stokes wave model decrease with increase in height. Figure 29 shows that mooring line tension decrease with cage diameter and also decreases with water depth which is corroborated by Huang et. al [12].

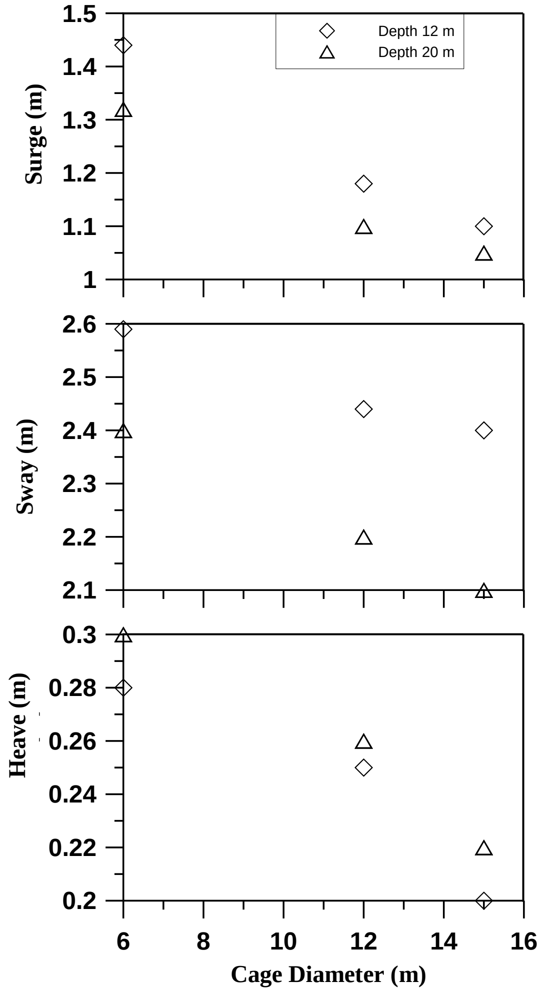

Figure 27. Response of cage against cage diameter for the 5 degree of freedom Stokes wave model

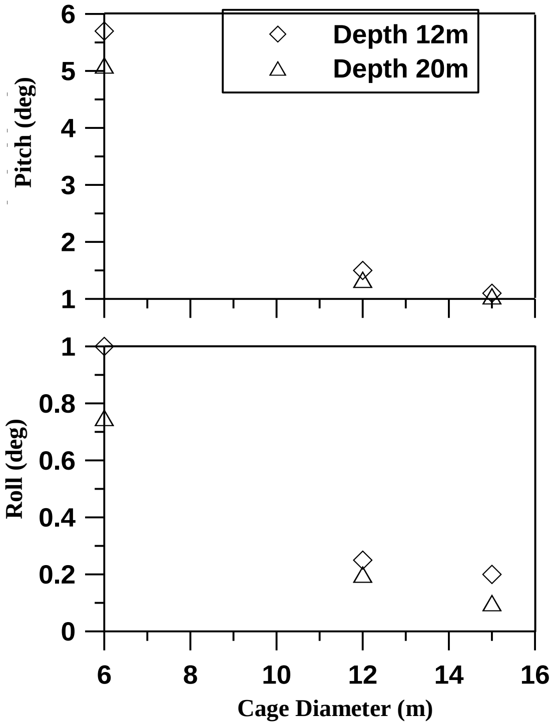

Figure 28. Angular Response of cage against cage diameter for the 5 degree of freedom Stokes wave model

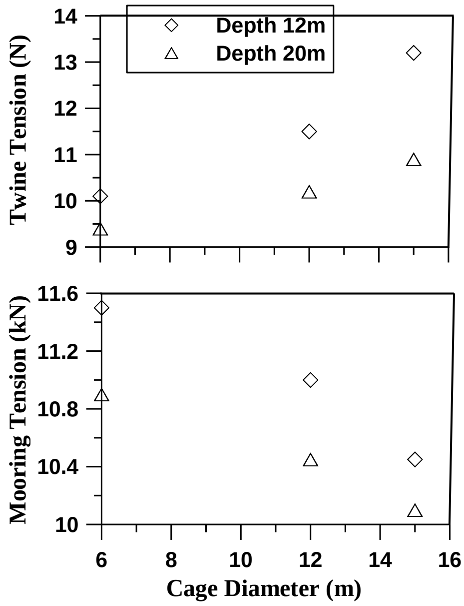

Figure 29. Mooring and twine tension of cage against cage diameter for the 5 degree of freedom Stokes wave model

5. Conclusions

In this paper, a 5 degree of freedom model for a floating coastal aquaculture cage has been developed. The estimation of stiffness, damping and wave exciting force on various cage components has been developed. Both linear wave and Stokes 5th order wave have been considered for computing wave forces on the cage. The main conclusions of this paper are as follows:

1. For developing a realistic model we need to consider a wave coming from any direction. Therefore a five degree of freedom model is developed along with an independent hanging mass. There is a reduction in motions due to the fact that energy of the wave is split into x and y directions, only heave motion shows the same pattern as the 3 degree of freedom model case. When the net and sinker are considered as a separate hanging mass the motions vary considerably with the net itself having significant motions which in turn reduces the mooring tension force, this is closer to the realistic situation as the net is independent to move.
2. The hanging mass motions show a sinusoidal and periodic pattern because the motions are uncoupled and the exciting forces are sinusoidal, though they still are affected by collar motions.
3. In Stokes 5th order theory though the wave force has increased but the motions show considerable reduction because of the change in the wave pattern and slope.
4. The mooring tensions have been determined for all cases. The mooring tension decrease with increase in wave period, cage diameter and water depth. The mooring line tension is within normal levels for the case investigated as the tension is below the failure limit for the given mooring line.

Acknowledgment

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List of Symbols

A	area of cross section
C_D	the drag coefficient
C_L	the lift coefficient
D_s	diameter of sinker pipe
$\hat{e}_n$	unit vector in normal direction
$\hat{e}_t$	unit vector in tangential direction
g	acceleration due to gravity
k	wave number
K_M	added mass coefficient
m	mass of cage
P_c	sum of perimeters of collar pipes
P_s	perimeter of sinker pipe
V	water particle velocity
U	current velocity in x direction
$\dot{R}$	cage or body velocity
α	angle of attack for drag coefficient
ρ	density of sea water
ω	angular frequency of wave

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