

# Reduction of Effect of Waves Disturbance on Ship Roll Motion by Using Robust Nonlinear $H_\infty$ Control

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## ABSTRACT

This paper presents a new robust fin control method based on  $L_2$  gain design to reduce ship roll motion and waves disturbance effects. The process involves the nonlinear dynamics of the ship roll and its fin actuator. External disturbances of waves and model parameters variations are considered as process uncertainties. The state feedback controller is designed to reduce the effect of wave disturbance on the controllable variables of the process. For controller design, nonlinear  $H_\infty$  approach has been employed in the sense that it ensures the stability of the process and achieves the desired control objectives. To solve the inequalities associated with the nonlinear  $H_\infty$  problem, an efficient SOSTOOL-based algorithm is presented. Simulation results are presented to evaluate the efficiency of the proposed approach. In order to validate the proposed algorithm, it is compared with PID controller. Results show the superiority of the designed controller based on Robust Nonlinear  $H_\infty$  control over PID controller.

## 1. Introduction

Each ship has three degrees of translational motions and three degrees of rotational motions. The heave, pitch and roll motions are rotational; because they are affected by hydrodynamic moment due to the interaction between fluid and ship. Among them, roll motion is of great importance; because its containment has a great impact on the stability of the ship, as the vertical acceleration, which is induced by roll motion of the ship, it can cause sea sickness for passengers and crew. Roll acceleration may produce cargo damages. On the other hand, large roll angles limit the installation of equipment such as weapons, balance systems, sonars, etc. The other reasons for the importance and application of roll motion can be found in [1, 2]. In most cases, the ship experiences every six motions at a time, but for the reasons mentioned, roll motion is more important. Many stabilization systems for roll motion have been introduced in the last several decades [3]. Equipped roll motion stabilizer can well guarantee ship performance in conditions where the ship is subject to adverse environmental impacts. Also, other different methods and tools have been introduced to reduce undesirable roll motion [4]. For example, in [5,6], of fin stabilizers used to reduce the rolling and heeling during ship turning. The fin stabilizer is considered as the most effective anti-roll technique for high speed ships, for the reasons detailed in [7]. Also, it is necessary for the stabilizer to reduce the roll motion

based on roll angle and roll rate, through control of the mechanical fin angle [8]. To reduce the roll motion in irregular waves, a LQG-based fin stabilizer is proposed in [9]. Also, a model predictive control (MPC) is proposed for ships affected by dynamic stall [1]. In active stabilizer design, controller design plays an important role in the reduction of roll motion. Hitherto, many stabilizer-based control strategies have been proposed, including conventional PID controller and many advanced controller designs. Most ships can be controlled by the PID, which takes advantage of the extended classical control theory [10]. In dealing with nonlinearities, unknown uncertainties and environmental disturbances in the roll dynamic system, some of the advanced controllers have shown a better damping performance than conventional methods. For example, in [11], a neural network controller is designed to identify nonlinearities in the dynamics of the loops. In [12], the fuzzy logic approach is used to obtain a robust fin controller. Also, in [13], a functional-link neural network is proposed to reduce roll motion. In [14], a first order sliding mode controller has achieved good results in stabilizing the dynamic position of roll motion.

In this paper, to cover the effects of uncertainties and disturbances, a novel robust  $H_\infty$  control method is proposed for nonlinear dynamics of the ship roll system. Due to the difficulty in solving Hamilton-Jacobi inequalities arising from the nonlinear  $H_\infty$  problem, this inequality is converted to a sums of

squares (SOS) problem that can be solved using the SOSTOOL plug-in of MATLAB software. Also, operational constraints of the model are included in the problem. Validation of the proposed algorithm is done through its comparison with PID controller.

The paper is continued as follows. Section 2 presents the ship roll model, which is provided to design the controller. In section 3, presents the fin actuator model. Section 4 briefly explains how the PID controller was designed. Section 5 concentrates on the design of nonlinear robust  $H_\infty$  controller for fin - roll Model in the presence of waves. In section 6, simulation results and discussion illustrate the effectiveness of the proposed algorithm, compared to PID controller. In the end, the contributions of the study are explained in Section 7.

## 2 Dynamics of ship roll system

The motion of a ship has six degrees of freedom (6DOF). Thus, as to the description of its motion, three coordinates have to be separately considered to define translations and orientation. As illustrated in Figure. 1, the definition of these coordinates is done by using two type of reference frames: inertial frames and body-fixed frames.

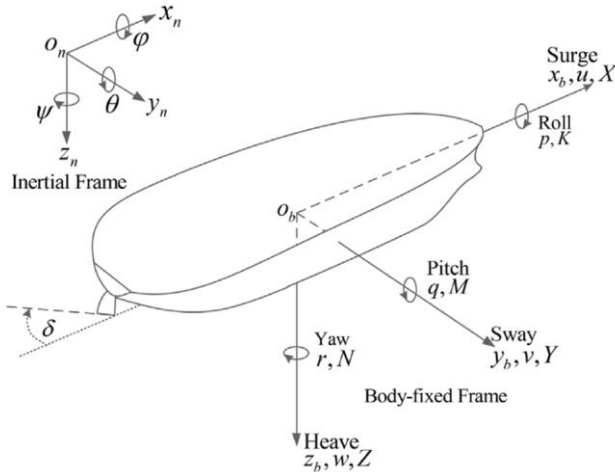


Figure 1. Using reference frames for describing ship motion

The description of ship model is generally as follows [3]:

$$\dot{\eta} = J(\eta)v \quad (1)$$

$$M_{RB}\dot{v} + C_{RB}v = \tau_d - \tau_f - \tau_h$$

Where the generalized displacement, body-fixed velocities and the hydrodynamic, control and disturbance forces are represented as the vector variables  $\eta$ ,  $v$ ,  $\tau_h$ ,  $\tau_c$  and  $\tau_d$  respectively. The matrices  $M_{RB}$  and  $C_{RB}$  refer to the rigid-body mass and the Coriolis-centripetal.

The production of small coupling between the roll and yaw motions occurs owing to the location of the fins in the hull and the center of gravity. Due to this negligible coupling, designing fin stabilizers

controllers can be done under the assumption that decoupling roll motion from the other motions is performed. This assumption simplifies the design and findings of classical controllers such as PID and  $H_\infty$ .

Otherwise, if the fins are located either far aft or ahead of the center of gravity, there will be a coupling with the steering, which can reduce the performance of the fins.

Thus, it can be assumed that the fins are located approximately in the middle of the hull close to the center of gravity and there is the least negligible coupling with other motions.

Hence, the dynamical model of the ship roll system is as follows [3]

$$I_{xx}\ddot{\phi} = \tau_d - \tau_f - \tau_h \quad (2)$$

$$p = \dot{\phi}$$

In the equation,  $\phi$  is the roll angle and  $p$  is the roll rate,  $\tau_f$  is moment created by fins,  $\tau_d$  is wave excitation moment and  $\tau_h$  is hydrodynamic moment in fluid and ship interactions and  $I_{xx}$  is roll moment of inertia. Subsequently, these moments are calculated with the data given in [3] for the corresponding ship. The fins moment is in the form of relation (3):

$$\tau_f = \rho V^2 A_f R_f C_L(\alpha_e) \quad (3)$$

Where,  $\rho$ ,  $V$ ,  $A_f$ ,  $R_f$  and  $C_L(\alpha_e)$  are the density of the flow, ship speed, fin area, fin lever, lift coefficient, respectively.

The torque resulting from the interaction between fluid and ship is obtained from equation (4) [2]:

$$\tau_h = K_p \dot{p} + f_1(\phi, \dot{\phi}) + f_2(\phi) \quad (4)$$

Where  $K_p \dot{p}$  term, regards a hydrodynamic moment in roll because of pressure variation that is proportionate to the roll accelerations, and the coefficient  $K_p$  is called roll added mass.  $f_1(\phi, \dot{\phi})$  is damping term and it can be expressed by equation (5).

$$f_1(\phi, \dot{\phi}) = k_p p + k_{p|p|} p |p| \quad (5)$$

where  $k_p p$  is a linear damping term, which includes forces due to wave making and linear skin friction, and the coefficient  $k_p$  is denoted a linear damping coefficient.  $k_{p|p|} p |p|$  is a non-linear damping term, which contains moments due to viscous effects, alike non-linear skin friction and eddy making due to flow separation, and the coefficient  $k_{p|p|}$  is denoted a non-linear damping coefficient.  $f_2(\phi)$  is the restoring moment term because of gravity and buoyancy, and it can be written as

$$f_2(\varphi) = \Delta GZ(\varphi) \quad (6)$$

where  $\Delta$  is the ship's displacement and  $GZ(\phi)$  is the restoring moment arm and it is the function of the roll angle. The  $GZ$  curve is an odd function and therefore represent with odd order polynomial. The descriptor polynomial is in the form of relation (7) [15]:

$$GZ(\varphi) = c_1\varphi + c_2\varphi^2 + c_5\varphi^5 \quad (7)$$

The coefficients are defined as (8):

$$c_1 = \frac{d(GZ)}{d\varphi} = GM$$

$$c_3 = \frac{4}{\varphi_v^4} (3A_{\varphi v} - GM\varphi_v^2) \quad (8)$$

$$c_5 = -\frac{3}{\varphi_v^6} (4A_{\varphi v} - GM\varphi_v^2)$$

Where  $GM$ ,  $\phi_v$  and  $A_{\varphi v}$  are, the metacentric height, angle of vanishing stability and area under the  $GZ$  curve, respectively.

The wave exciting moment is specified as

$$\tau_d = \sum_{i=1}^n I_{xx} \omega_e^{(i)^2} \alpha_{\max} \cos(\omega_e^{(i)} t) \quad (9)$$

Where  $\alpha_{\max}$  is the maximum of wave slope and  $\omega_e$  is the wave encounter frequency.

In this paper, the performance of the proposed algorithm is illustrated via a fishery ship. Table 1 shows the main ship parameters.

**Table 1. Main parameters of the ship**

Characteristic	Value
Length	84 [m]
Width	10 [m]
Draft	3.2 [m]
Displacement	1300 [t]
Transverse metacenter height	1[m]
Roll period	8.5 [s]
Dimensional decay coefficient	0.1215

The hydrodynamic coefficients for the fishery ship are expressed in Table 2. Hydrodynamic coefficients were extracted according to [16].

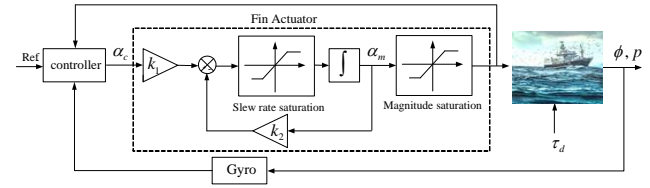
**Table 2. The calculated coefficients for the fishery ship**

Characteristic	Value
$c_1$	$0.63 [kg.m^2.s^{-2}]$
$c_3$	$-0.0406 [kg.m^2.s^{-2}]$
$c_5$	$0.2102 [kg.m^2.s^{-2}]$
$k_p$	$9.35 [kg.m^2.s^{-1}]$

$I_{xx} + k_p$	$63.25 [kg.m^2]$
$k_{p p }$	$0.3975 [kg.m^2]$

### 3 Dynamics of fin actuator model

Electro-hydraulic servomechanism is used as an active fin actuator. Figure. 2 summarizes block diagram of the fin-roll closed loop control system. Outputs of the roll system are roll angle and roll rate, which are measured by a gyroscope and fed to the controller. The servo command output with an electromechanical mechanism is employed to fin actuator in order to achieve desired conditions.



**Figure 2. Block diagram of the fin-roll closed loop control system**

In different references, the first-order dynamics is considered as the relation (10) for fin actuator:

$$T_e \dot{\alpha}_m + \alpha_m = K_{dc} \alpha_c \quad (10)$$

Where  $\alpha_c$  is input variable of fin's actuator as fin angle command and  $\alpha_m$  is output variable as mechanical fin angle. Also  $K_{dc}$  is dc gain of the actuator and  $T_e$  is time constant due to the delay between  $\alpha_c$  and  $\alpha_m$ . The two operating constraints are considered for this system. In this paper  $K_{dc} = 1$  and  $T_e = 1s$ , are considered. Table 3 indicates the principle parameters of the fin stabilizer.

**Table 3. Main parameters of the fin stabilizer**

Characteristic	Value
Area	$4 [m^2]$
Chord	$2.17 [m]$
Aspect ratio	0.85
Roll arm	$5.7 [m]$
Lift coefficient	0.055

### 4- Summary of PID controller Design

As to demonstrating the efficiency of the proposed algorithm, the PID controller, which is commonly and practically used to control fin stabilizers, is designed. The PID controller is usually stated below, which involves three control gains, i.e.  $K_p$ ,  $K_I$  and  $K_D$ :

$$y(t) = K_p e(t) + K_I \int_0^t e(t) dt + K_D \frac{de(t)}{dt} \quad (11)$$

where the control output and the system error are represented by  $y(t)$  and  $e(t)$ .

Eq. (11) can be rewritten to control fin stabilizer:

$$\alpha_f(t) = K_p \dot{\phi}(t-1) + K_I \phi(t-1) + K_D \ddot{\phi}(t-1) \quad (12)$$

where  $\alpha_f(t)$  is the control output fin angle,  $\phi(t-1)$ ,  $\dot{\phi}(t-1)$  and  $\ddot{\phi}(t-1)$  are the roll angle, roll rate and roll acceleration, respectively.

The designed PID controller optimal parameters are  $K_p=1.2605$ ,  $K_I=0.6316$  and  $K_D=0.6148$ , which are extracted by Monte Carlo Simulation (MCS) optimization algorithm.

## 5- Design of nonlinear robust $H_\infty$ controller for ship roll model

### 5-1- Definitions and Preliminaries

In this section, robust  $H_\infty$  nonlinear control problem and the sum of squares (SOS) problem are introduced as the basis for this design.

An uncertain affine nonlinear system (13) is considered, in which  $z$  is the controlled output of the system.

$$\begin{aligned} \dot{x} &= F(x) + \Delta F(x, \theta) + g_1 w + g_2 u \\ z &= \begin{bmatrix} h_1(x) \\ u \end{bmatrix} \end{aligned} \quad (13)$$

Also,  $\Delta F(x, \theta)$ ,  $\theta \in \mathcal{G}$  are system uncertainty and uncertain parameter, respectively. To define and solve the  $H_\infty$  control problem, assumptions should be considered on system (13).

Assumption1:  $w$  in the interval  $[0, T]$  is  $L_2$  signal ( $w \in L_2[0, T]$ ).

Assumption 2: Acceptable sets for system uncertainty and uncertain parameters are as follows.

$$\begin{aligned} \mathcal{G} &= \{ \theta \mid |\theta| \leq \theta_u \} \\ \mathcal{S} &= \left\{ \begin{array}{l} F[F(x, \theta) = H_2(x)F(x, \theta)E_1(x)] \\ F(x, \theta)^2 \leq 1, \forall x \in R^n, \theta \in \mathcal{G} \end{array} \right\} \end{aligned} \quad (14)$$

Since  $L_2$  gain is used in the definition of  $H_\infty$  problems, assumption 1 has been considered. Assumption 2 is a condition of well-known compatibility, and the uncertainties that satisfy this condition are called structural uncertainties.

In the control of  $H_2$ , the goal is to stabilize the system and reduce the effect of external disturbances on controllable variables.

In order to reduce the disturbance effect, the  $L_2$  gain is used, in which case the nonlinear robust  $H_\infty$  control problem can be defined for system (14).

**Definition1:** (nonlinear robust  $H_\infty$  control problem with state feedback): if there is, find the control law  $u = \alpha(x)$ , such that the closed loop system (15) is internal stable for all  $\Delta F(x, \theta)$ ,  $\theta \in \mathcal{G}$ . Also, the gain of the system from the input  $w$  to the output  $z$  is less than or equal to the constant number of the  $\gamma > 0$  [17].

$$\begin{aligned} \dot{x} &= F(x) + \Delta F(x, \theta) + g_1 w + g_2 \alpha(x) \\ z &= \begin{bmatrix} h_1(x) \\ \alpha(x) \end{bmatrix} \end{aligned} \quad (15)$$

A necessary and sufficient condition for solving this problem is the existence of a positive (semi) definite  $V(x)$  for the Hamilton-Jacobi-Isak inequality (16). Also the solution to the problem is the control law in the form of relation (17) [17].

$$V_x f(x) + \frac{1}{2} V_x \left( \frac{1}{2\gamma^2} g_1 g_1^T + H_2(x) H_2^T(x) - 2g_2 g_2^T \right) V_x^T + \quad (16)$$

$$\begin{aligned} \frac{1}{2} h_1^T(x) h_1(x) + \frac{1}{2} E_1^T(x) E_1(x) &\leq 0 \\ u &= -g_2^T V_x^T \end{aligned} \quad (17)$$

In general, there is no method for obtaining the response of inequality (16). This has limited the use of nonlinear  $H_\infty$  control, as compared to its linear version. However, solutions are provided for certain classes of systems. One of these solutions is the use of the SOSTOOLS plug-in of MATLAB software [18], which is suitable for systems with nonlinear polynomial agents such as ship roll systems. Non-negative investigation of a multivariable function is a fundamental issue that appears in many fields of mathematics and control. In fact, the goal is to find the equivalent conditions and to provide an approach to validate the inequalities such as (16). A polynomial function  $F(x)$  is said to be nonnegative or positive (semi), if this function is greater than or equal to zero for all  $x \in R^n$  [19]. Clearly, in order for a polynomial function to be positive (semi) definite, it is necessary that the general degree be even.

The function  $F(x)$  is said as the sum of squares (SOS), if there are  $m_1(x), m_2(x), \dots, m_p(x)$  polynomials so that  $F(x)$  can be written in terms of the sum of

$$\text{squares of these functions } M(x) = \sum_{i=1}^p m_i(x) \quad [19].$$

$$M(x_1, x_2, \dots, x_n) > 0, \forall x_1, x_2, \dots, x_n \quad (18)$$

The polynomial functions can be written in quadric for  $M(x) = x^{[d]T} Q x^{[d]}$ , in which the constant matrix  $Q$  is not unique. In this representation, if  $Q$  is positive definite, then  $M(x)$  will also be positive definite. It should be noted that in the quadratic form, the positive definite and *SOS* of  $M(x)$  are the same [19]. The *SOS* issue of a polynomial can be projected in the form of a convex optimization problem. MATLAB software SOSTOOLS plugin can solve such problems.

## 5-2 nonlinear robust $H_\infty$ control

In this section, the goal is to design a control law in the form (17) for the ship roll system. This will result in finding a positive definite solution for Hamilton-Jacobi's inequality (16). In this section, by providing a theorem and an algorithm, an efficient solution is presented to find the inequality response (16), taking into account the operational limitations of the ship roll system. The control objectives of the system include reducing the effect of sea wave disturbances on roll angle and roll rate outputs, as well as reducing control effort, so the controlled output is defined as  $z = [x_1 \ x_2 \ u]^T$

As mentioned above, in order to solve the inequality (16), it is converted into a *SOS* problem, which makes it possible to investigate the existence of its response by MATLAB.

**Theorem 1:** The nonlinear robust  $H_\infty$  control problem with state feedback will be solved for system (13) by using of control law (17), If for  $\gamma > 0$ , matrix inequalities  $M \pm 0$  is satisfied.

$$M = \begin{pmatrix} -2x^T P F(x) & h_1^T & E_1^T(x) & 2x^T P & 2x^T P g_1 \\ * & 2I & 0 & 0 & 0 \\ * & * & 2I & 0 & 0 \\ * & * & * & R_2^{-1} & 0 \\ * & * & * & * & 2\gamma^2 I \end{pmatrix}$$

$$R_2 = \frac{1}{2} (H_2(x) H_2^T(x) - 2g_2 g_2^T)$$

**Proof:** According to the explanations in Section 5-1, the necessary and sufficient condition for solving nonlinear robust  $H_\infty$  control problem with state feedback, leads to the existence of a definite positive solution for the Hamilton-Jacobi Nathasian (16). It must be shown that the matrix inequality  $M \pm 0$  arises from the inequality (16). With the choice of  $V(x) = x^T P x$ ,  $P = P^T \succ 0$  and using lemma Schur, It simply appears.

Note 1: By choosing the arbitrary vector  $S$  with the appropriate dimensions, we can express the matrix

$M \pm 0$  in the form of the *SOS* problem. Therefore, in order to solve the nonlinear robust  $H_\infty$  control problem with state feedback for system (15), it is sufficient to answer the following *SOS* problem.

$$SOSP : find \ P = P^T \succ 0 \ s.t. \ s^T M s \in \Phi_{SOS}$$

Note 2: If there are conditions as  $R_i(x) < 0$  on states, to find the local response, the *SOS* problem can be corrected as follows [20].

$$SOSP_{mod} : find \ P = P^T \succ 0$$

$$s.t. \ \left\{ s^T M s + \sum_{i=1}^n \lambda_i(x, \gamma) R_i(x) \right\} \in \Phi_{SOS}$$

Where  $\lambda_i(x, \gamma) > 0$ .

In the following, an algorithm for programming and solving the *SOS* problem is presented. In the algorithm, it attempts to minimize  $\gamma$  by repeating on it.

**Algorithm 1:** step1. Initialization  $\gamma_0$  step2. Solve the problem  $SOSP_{mod}$ ,  $i = i + 1$  step3. If  $SOSP_{mod} \notin \Phi_{SOS}$ , then  $\gamma_i = \gamma_{i-1} + 0.1$  and back to step 2 step4. Otherwise  $\gamma_i = \gamma_{i-1} - 0.01$  step5. Solve the problem  $SOSP_{mod}$ ,  $i = i + 1$  step6. If  $SOSP_{mod} \in \Phi_{SOS}$ , go back to step 4 step7. Otherwise  $\gamma^* = \gamma_{i-1}$ ,  $u = -2g_2 P x$ .

According to the ship and fin specifications as well as the hydrodynamic coefficients, the uncertainty of system (15) is  $\Delta F(x, \theta) = [0, 0.006\theta x_2^2, 0]^T$

Where  $\theta \in \{-1, 1\}$  is an uncertain parameter. We can write  $\Delta F(x, \theta)$  as the following relation that satisfies the condition of compatibility.

$$\Delta F(x, \theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 10 \end{bmatrix} \theta \begin{bmatrix} 0 \\ 0.006x_2^2 \\ 0 \end{bmatrix} \quad (19)$$

Now, by choosing the state variables as  $[x_1 \ x_2 \ x_3]^T := [\varphi \ p \ \alpha_m]^T$  And also,  $d := T_e$  and  $u := \alpha_c$ ,

we can obtain the state space in form (20).

$$\begin{aligned} \dot{x}_1 &= F_1(x) \\ \dot{x}_2 &= F_2(x) + \Delta F_2(x, \theta) + 3.41 \times 10^{-8} d \\ \dot{x}_3 &= F_3(x) + 0.5u \end{aligned} \quad (20)$$

Where



$$F_1(x) = x_2$$

$$F_2(x) = -0.64x_1 - 0.02x_2 + 0.34x_3 + 0.31x_1^3 - 0.1x_1^5$$

$$F_3(x) = -0.5x_3$$

$$\Delta F_2(x, \theta) = -0.006\theta x_2^2, \quad \theta \in \{-1, 1\}$$

## 6- Simulation results

The sailing condition is assumed for a random sea at the forward speed 20 knots. The significant wave height is selected to 3 m and the average period of the wave is set to 8.5 s.

Also according to the ship data,  $\alpha_{sat}$  and  $\alpha_{stall}$  are 30 and 35 degrees respectively, applying such constraints to the system states.

$$-18.75^0 < x_2 < 18.75^0, \quad -30^0 < x_3 < 30^0$$

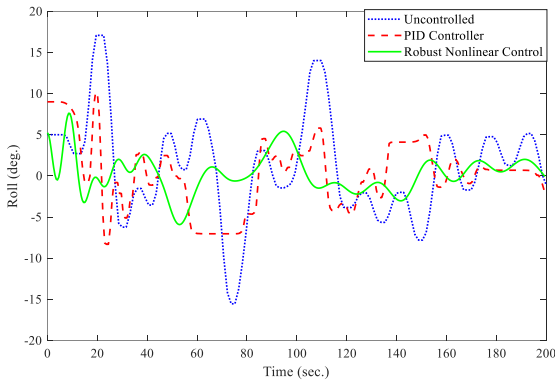
Based on Algorithm 1, the controller design parameters are selected as follows.

$$\gamma^* = 1.01$$

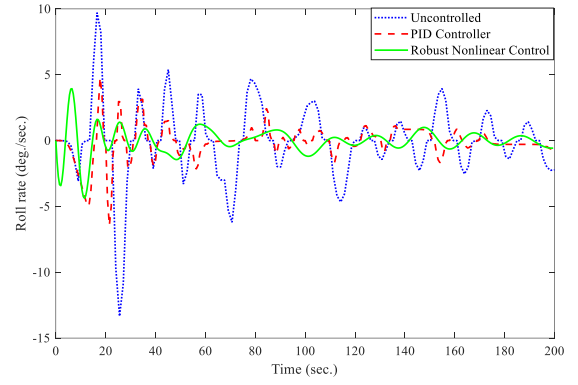
$$P = 10^8 \begin{bmatrix} 9.01 & 0.13 & 0.0002 \\ 0.13 & 0.17 & 0.0003 \\ 0.0002 & 0.0003 & 75 \times 10^{-6} \end{bmatrix}$$

$$w_* = \begin{cases} w & t < 50 \\ 0 & t \geq 50 \end{cases}$$

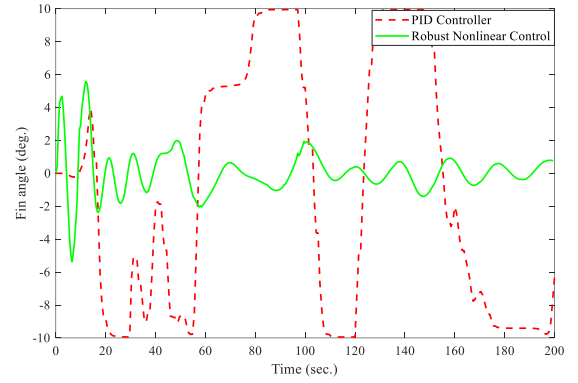
The simulation results of ship roll motion at encounter angle of  $45^\circ$ ,  $90^\circ$  and  $135^\circ$  are shown in Figures. 3, 4 and 5, respectively. The roll reduction performance of the two controllers are shown in Table 4.



(a) Roll angle

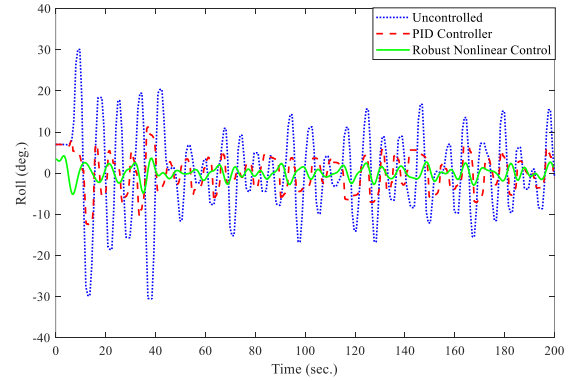


(b) Roll rate

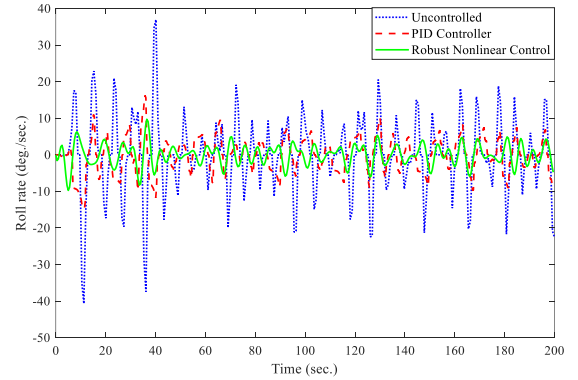


(c) Fin angle

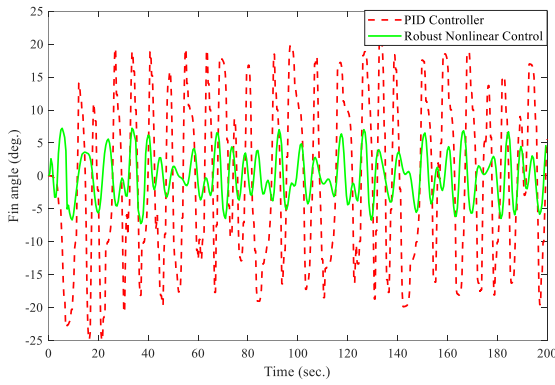
Figure 3. Ship roll motion at encounter angle of  $45^\circ$



(a) Roll angle

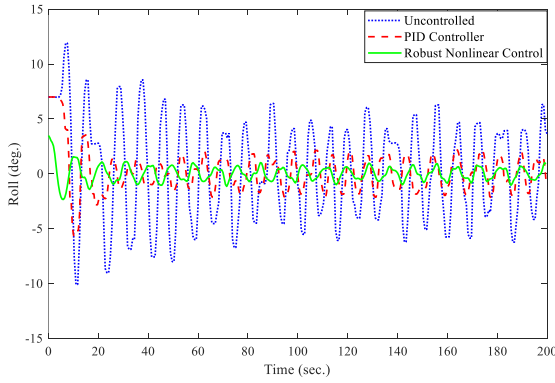


(b) Roll rate

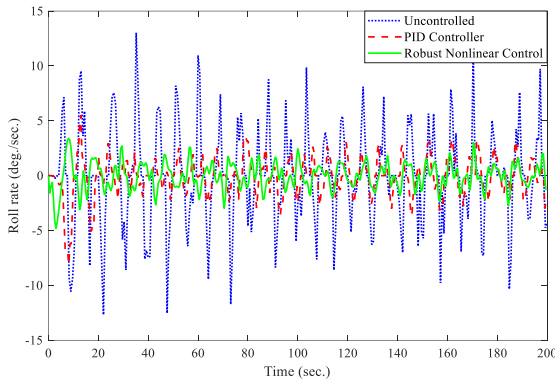


(c) Fin angle

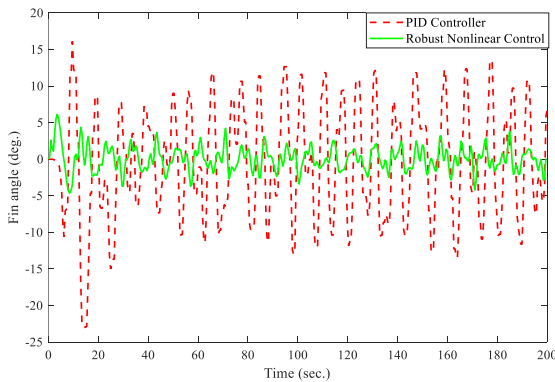
Figure 4. Ship roll motion at encounter angle of 90°



(a) Roll angle



(b) Roll rate



(c) Fin angle

Figure 5. Ship roll motion at encounter angle of 135°

It can be seen from Figures. 3, 4 and 5 and Table 4 that both the two controllers can reduce ship roll motion at different encounter angles. However, the roll reduction effect of nonlinear robust  $H_\infty$  control are better than that of the conventional PID controller. The anti-rolling effects of the nonlinear robust  $H_\infty$  controller at the three encounter angles are all above 73%. The anti-rolling effect of nonlinear robust  $H_\infty$  controller at encounter angle of 90° is up to 87.2%, which can be considered very satisfactory. While, the anti-rolling effect of the conventional PID controller at encounter angle of 90° is 75%, which is also satisfactory. However, the roll-reduction effect of the PID controller at encounter angle of 45° is only 57.6%, which is not satisfactory. From Figures 3, 4 and 5, we can see that the fin angle under nonlinear robust  $H_\infty$  control is relatively small than that under PID control, and that may be account for the difference in anti-rolling effect. According to the simulation results, it can be concluded that the nonlinear robust  $H_\infty$  controller is more effective in ship roll reduction control using fin stabilizers, with higher anti-rolling effect than the PID controller at different encounter angles.

Table 4. Comparison of the performance of controllers

Encounter angle (deg.)	Control method	Roll angle (deg.)	Anti - rolling effect %
45	No control	12.5	-
	PID control	5.3	57.6
	Robust control	3.3	73.6
90	No control	17.2	-
	PID control	4.3	75
	Robust control	2.2	87.2
135	No control	9.2	-
	PID control	2.5	72.8
	Robust control	1	89.1

## 7- Conclusion

In this paper, a nonlinear robust  $H_\infty$  controller was designed to stabilize and reduce the effect of waves disturbance on ship roll motion. The SOS approach was used to solve the inequality arising from the nonlinear  $H_\infty$  problem, due to the nonlinear polynomial structure of the system. In the meantime, the nonlinear attenuation damping caused by the viscosity effects had expressions other than standard polynomials that were considered to be system model uncertainties and because of such uncertainties, a robust approach was used. The operating constraints of the system were incorporated into the problem solving and an algorithm was introduced to solve the problem approaching the minimum  $\gamma$ . As for comparison, the conventional PID controller was designed with the optimal control parameters obtained

through Monte Carlo Simulation. The simulation of ship roll motion at encounter angles of  $45^\circ$ ,  $90^\circ$  and  $135^\circ$  was carried out. The simulation results showed that nonlinear robust  $H_\infty$  controller via fin stabilizer was able to significantly reduce the effect of external disturbances on system output despite operational constraints.

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