

# Degree of Bending (DoB) in Tubular KT-Joints of Jacket Structures Subjected to Axial Loads

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## ABSTRACT

The fatigue life of tubular joints commonly found in offshore industry is not only dependent on the value of Hot-spot stress (HSS), but is also significantly influenced by the through-the-thickness stress distribution characterized by the degree of bending (DoB). The determination of DoB values in a tubular joint is essential for improving the accuracy of fatigue life estimation using the stress-life (S–N) method and particularly for predicting the fatigue crack growth based on the fracture mechanics (FM) approach. In the present paper, data extracted from finite element (FE) analyses of tubular KT-joints, verified against experimental data and parametric equations, was used to investigate the effects of geometrical parameters on DoB values at the crown 0°, saddle, and crown 180° positions along the weld toe of central brace in tubular KT-joints subjected to axial loading. Parametric study was followed by a set of nonlinear regression analyses to derive DoB parametric formulas for the fatigue analysis of KT-joints under axial loads. The tubular KT-joint is a quite common joint type found in steel offshore structures. However, despite the crucial role of the DoB in evaluating the fatigue performance of tubular joints, this paper is the first attempt to study and formulate the DoB values in KT-joints.

## 1. Introduction

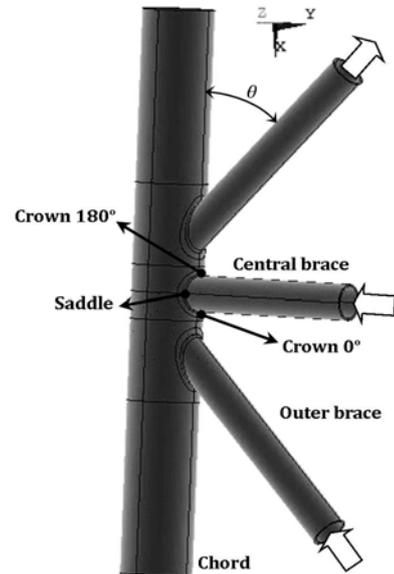
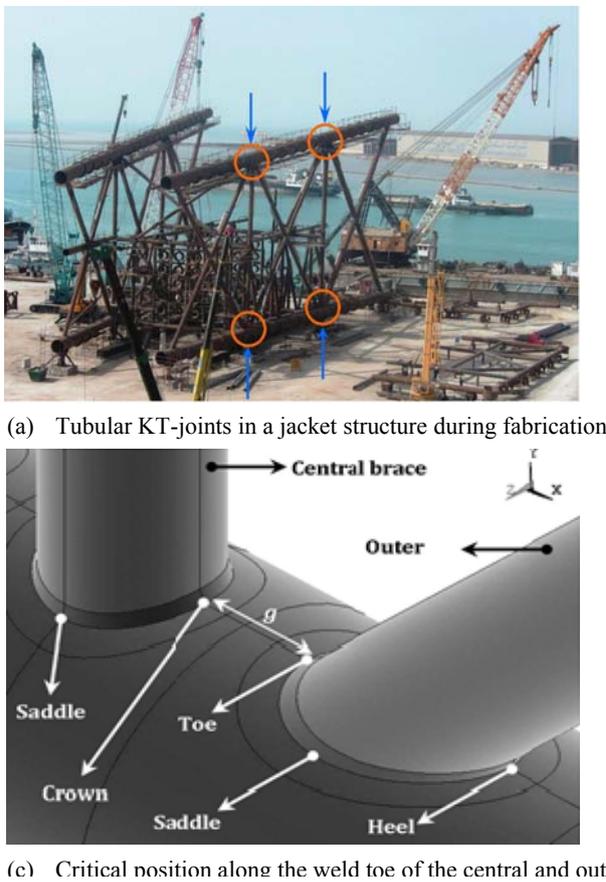
Offshore jacket structures are composed of circular hollow section (CHS) members, also called tubulars. The intersection between tubulars, in which one or more branch members (braces) are welded to the undisturbed surface of the main member (chord), is called a tubular joint Figure 1 Tubular joints are exposed to cyclic loads due to sea waves and hence are critical locations with respect to fatigue induced damage. The stress-life (S–N) method, based on the hot-spot stress (HSS), is widely used to assess the fatigue life of a tubular joint. However, the study of a large number of fatigue test results have shown that tubular joints of different geometry or loading type but with similar HSSs often exhibit significantly different numbers of cycles to failure [1]. These differences are thought to be attributable to changes in crack growth rate which is dependent on the through-the-thickness stress distribution characterized by the degree of bending (DoB), i.e. the ratio of bending stress over total external stress, ratio expressed as:

$$\text{DoB} = \frac{\sigma_B}{\sigma_T} = \frac{\sigma_B}{\sigma_B + \sigma_M} \quad (1)$$

Where  $\sigma_T$  is the total stress;  $\sigma_B$  and  $\sigma_M$  are the bending stress and membrane stress components, respectively. Figure 2 depicts a typical stress distribution through the chord wall of a tubular joint. It can be observed that the through-the-thickness stress field is predominantly due to the linear chord wall bending and the non-linear stress concentration at the weld toe due to the section change at the intersection. The non-linear distribution around the weld-toe stress concentration region is dependent on the weld toe geometry and is difficult to predict during the design stage. However, it would have little effect for a deep crack [2]. Thus, the stress distribution across the wall thickness is assumed to be a linear combination of membrane and bending stresses. As mentioned before, it has become evident that the HSS is not enough to characterize all aspects of fatigue failure. Therefore, the standard stress-life approach may be unconservative for the joints with low DoB. Hence, the current standard HSS-based S–N approach can be modified to include the effect of DoB representing the through-the-thickness stress distribution in the tubular joint in order to reduce the scatter in the S–N curve

and obtain more accurate fatigue life prediction. The other shortcoming of the S–N approach is that this method gives only the total life and can not be used to predict the fatigue crack growth and the remaining life of cracked joints. For the fatigue analysis of cracked joints, fracture mechanics (FM) should be used. The accurate determination of a stress intensity factor (SIF) is the key for FM calculations. Owing to the complexities introduced by the structural geometry and the nature of the local stress fields, it is impossible to calculate the SIFs analytically. This problem is often tackled by using the simplified models, such as the flat plate solution and methods based on the T-Butt weight function with an appropriate load shedding model. In order to use these simplified SIF models to calculate the remaining fatigue life of tubular joints, the information is required again on the distribution of through-the-thickness stress acting on the anticipated crack path, which can be characterized by the DoB. Thus, the DoB is an important input parameter for the calculation of fatigue crack growth in tubular welded joints. Under any specific loading condition, the DoB value along the weld toe of a tubular joint is mainly determined by the joint geometry. To study the behavior of tubular joints and to easily relate this behavior to the geometrical characteristics of the joint, a set of dimensionless geometrical parameters has been defined. Figure 1 depicts a tubular KT-joint with the geometrical parameters  $\tau$ ,  $\gamma$ ,  $\beta$ ,  $\zeta$ ,  $\alpha$ , and  $\alpha_B$  for

chord and brace diameters  $D$  and  $d$ , and their corresponding wall thicknesses  $T$  and  $t$ . Critical positions along the weld toe of the brace/chord intersection for the calculation of DoB in a tubular joint, i.e. saddle, crown, toe and heel, have been shown in Figure 1. The tubular KT-joint is a quite common joint type found in steel offshore structures. However, despite the crucial role of DoB in evaluating the fatigue performance of tubular joints, DoB values in KT-joints have not been investigated so far and no design equation is available to determine DoB values for joints of this type. In this paper, results of numerical study of the degree of bending in tubular KT-joints are presented and discussed. In this research program, a set of parametric stress analyses was performed on 46 KT-joint finite element (FE) models subjected to axial loading (Figure 1). The analysis results were used to present general remarks on the effect of geometrical parameters including  $\tau$  (brace-to-chord thickness ratio),  $\gamma$  (chord wall slenderness ratio),  $\beta$  (brace-to-chord diameter ratio), and  $\theta$  (outer brace inclination angle) on DoB values at the crown  $0^\circ$ , saddle, and crown  $180^\circ$  positions along the weld toe of the central brace. Based on results of KT-joint FE models which were verified as discussed in Section 3.5, a DoB database was prepared and a new set of DoB parametric equations was established, based on nonlinear regression analyses, for the fatigue analysis and design of KT-joints subjected to axial loading.



$D$ : Chord diameter;  $d$ : Brace diameter  
 $T$ : Chord wall thickness;  $t$ : Brace wall thickness  
 $L$ : Chord length;  $l$ : Brace length;  
 $g$ : Gap

$\beta = d/D$ ;  $\gamma = D/2T$ ;  $\tau = t/T$   
 $\zeta = g/D$ ;  $\alpha = 2L/D$ ;  $\alpha_B = 2l/d$

Figure 1. Geometrical notation for a tubular KT-joint subjected to axial loading

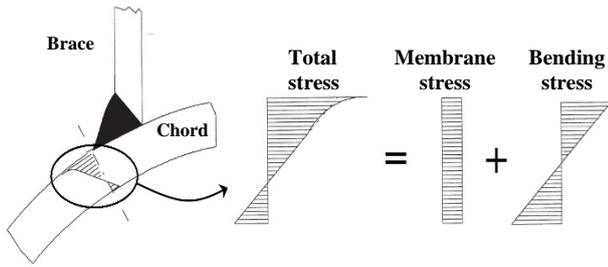
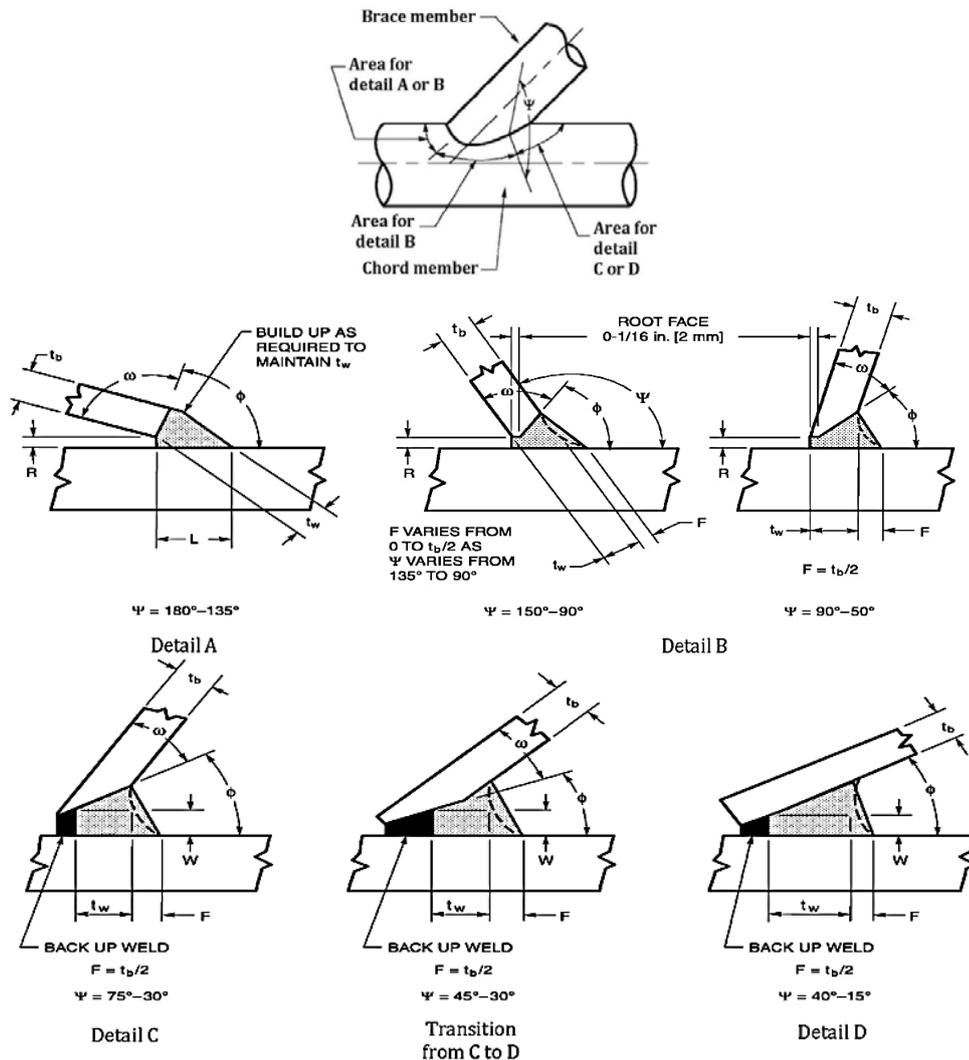


Figure 2. Through-the-thickness stress distribution in a tubular joint [2]

**2. Literature Review**

Morgan and Lee [3] derived mean and design equations for DoB values at critical positions in axially loaded tubular K-joints from a previously established FE database of 254 joints. Design equations met all the acceptance criteria

recommended by the UK DoE [4]. Chang and Dover [2] carried out a series of systematic thin shell FE analyses for 330 tubular X- and DT-joints typically found in offshore structures, under six different types of loading. Based on the results of nearly 2000 FE analyses, a set of parametric equations was developed to calculate the DoB at critical positions. Lee and Bowness [5] proposed an engineering methodology for estimating SIF solutions for semi-elliptical weld-toe cracks in tubular joints. The methodology uses the T-butt solutions proposed previously by the authors in conjunction with the stress concentration factors (SCFs) and the DoB values in uncracked tubular joints.



Parameter	Detail A Ψ=180°-135°	Detail B Ψ=150°-90°	Detail C Ψ=75°-30°	Detail D Ψ=40°-15°
Root opening (R)	1.6-5 mm	1.6-6 mm	3-13 mm	3-13 mm
Joint included angle (φ)				
Max	90°	90° for Ψ ≤ 150°	40°; if more use Detail B	
Min	45°	37.5°; if unless use Detail C	Ψ/2	
Complete weld				
tw	≤ t1	≥ t1 for Ψ ≥ 90°	≥ t1/sinΨ but ≤ 1.75 t1	
Lw	≥ t1/sinΨ but ≤ 1.75 t1	≥ t1/sinΨ for Ψ ≤ 150°	Weld may be built up to meet this	≥ 2t1

Figure 3. Definition of the dihedral angle and details of the weld profile according to AWS D 1.1 [23]

Shen and Choo [6] determined the SIFs for a grouted tubular joint. They found that the fatigue strength of grouted joint may be lower than that of as-welded joint, because when normalized with the HSS, the shape factor of grouted joint is higher than that of original as-welded joint due to the reduction in the DoB caused by the presence of in-filled grout in the chord. For grouted tubular joints, it is essential to consider the effect of the DoB in practical fatigue assessment using HSS approach. Based on the above discussion, it can be concluded that despite the comprehensive research work accomplished on the investigation of SCFs and SIFs in tubular joints (e.g. [7-18] for SCFs, and [19-22] for SIFs), the research works on the DoB in tubular joints are scarce and the studied joint types and loading conditions are rather limited. Although the tubular KT-joints are commonly found in offshore jacket-type platforms, the DoB values in KT-joints under the axial loads have not been studied so far and no design equation is available to determine the DoB values for axially loaded joints of this type.

### 3. FE modeling and analysis

FE-based software package ANSYS was used in the present research for the FE modeling and analysis of tubular KT-joints subjected to axial loading in order to extract DoB values for the parametric study and formulation. This section presents the details of FE modeling and analysis.

#### 3.1. Weld profile

The accurate modeling of the weld profile is one of the important factors affecting the accuracy of the DoB results. In the present research, the welding size along the brace/chord intersection satisfies the AWS D 1.1 [23] specifications.

Details of the weld profile according to AWS D 1.1 [23] are shown in Figure 3. In this figure,  $L$  and  $t_w$  refer to the length and thickness of the weld, respectively; and  $\psi$  is the dihedral angle defined as the angle between the chord and brace surface along the intersection curve.

For more details of weld profile modeling, the reader is referred to Lie et al. [24] and Ahmadi et al. [25].

#### 3.2. Boundary conditions

Due to the XY-plane symmetry in the joint geometry and loading, only half of the entire KT-joint subjected to axial loading was required to be modeled (Figure 4). Displacements of the nodes on the symmetry plane were restrained perpendicular to the plane.

The end fixity conditions for the chord member of tubular joints in offshore structures may range from almost fixed to almost pinned with generally being closer to almost fixed [8]. In practice, value of the parameter  $\alpha$  in over 60% of tubular joints is in excess of 20 and is bigger than 40 in 35% of the joints [26].

Changing the end restraint from fixed to pinned results in a maximum increase of 15% in the HSS at crown position for  $\alpha = 6$  joints, and this increase reduces to only 8% for  $\alpha = 8$  [3]. In view of the fact that the effect of chord end restraints is only significant for joints with  $\alpha < 8$  and high  $\beta$  and  $\gamma$  values, which do not commonly occur in practice, both chord ends were assumed to be fixed, with the corresponding nodes restrained.

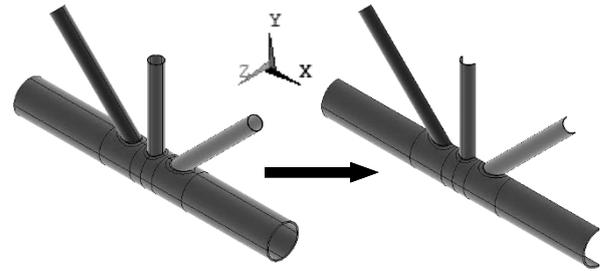


Figure 4. Half of the entire joint required to be modeled

#### 3.3. Mesh generation

ANSYS element type SOLID95 was used in the present study to model the chord, braces, and the weld profiles. These elements have compatible displacements and are well-suited to model curved boundaries. The element is defined by 20 nodes having three degrees of freedom per node and may have any spatial orientation. Using this type of 3-D brick elements, the weld profile can be modeled as a sharp notch. This method will produce more accurate and detailed stress distribution near the intersection in comparison with a simple shell analysis. In order to guarantee the mesh quality, a sub-zone mesh generation method was used during the FE modeling. In this method, the entire structure is divided into several different zones according to the computational requirements. The mesh of each zone is generated separately and then the mesh of entire structure is produced by merging the meshes of all the sub-zones. Quality and quantity of the mesh can be feasibly controlled by this method and badly distorted elements can be avoided. The mesh generated by this method for a tubular KT-joint is shown in Figure 5.

It is explained in Section 3.4 that the geometric stresses perpendicular to the weld toe are required to be calculated in order to determine the DoB at the weld toe position based on Eq. (1). As shown in Figure 5b, to extract the geometric stresses perpendicular to the weld toe, the region near the weld toe was meshed finely. The width of this region is discussed in Section 3.4. To verify the convergence of FE results, convergence test with different mesh densities was conducted before generating the 46 FE models for the parametric study.

**3.4. Analysis and DoB calculation**

Static analysis of the linearly elastic type is suitable to determine the DoB in tubular joints. For the chord, braces, and the weld profile, the Young’s modulus and Poisson’s ratio were taken to be 207 GPa and 0.3, respectively.

In order to determine the weld-toe DoB, according to Eq.(1), bending and membrane stress components should be known. These components can be calculated as follows:

$$\sigma_B = \frac{\sigma_O - \sigma_I}{2} \tag{2}$$

$$\sigma_M = \frac{\sigma_O + \sigma_I}{2} \tag{3}$$

where  $\sigma_O$  and  $\sigma_I$  are the hot-spot stresses (HSSs) at the weld toe on the outer and inner surfaces of the chord, respectively. Eqs. (1)-(3) lead to the following relation for the determination of DoB based on HSSs:

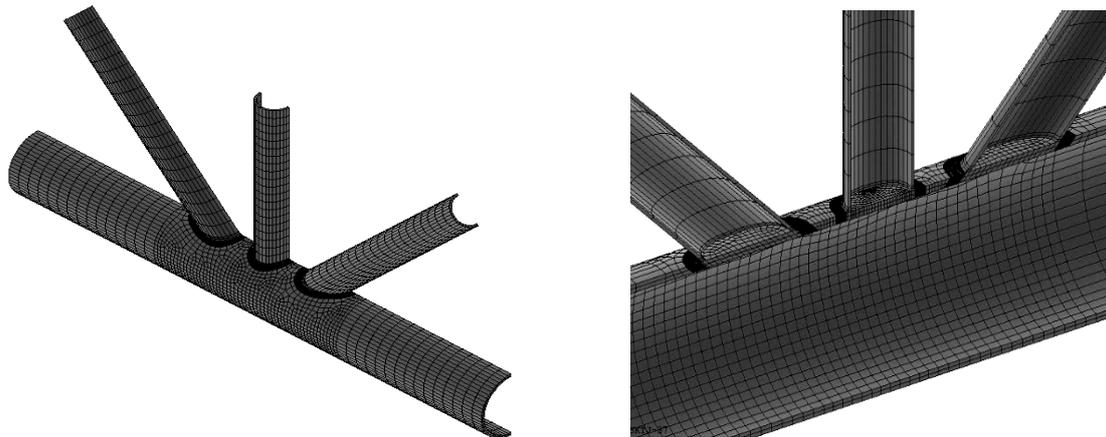
$$DoB = \frac{1}{2} \left( 1 - \frac{\sigma_I}{\sigma_O} \right) \tag{4}$$

To determine the HSSs, the stress at the weld toe position should be extracted from the stress field outside the region influenced by the local weld toe

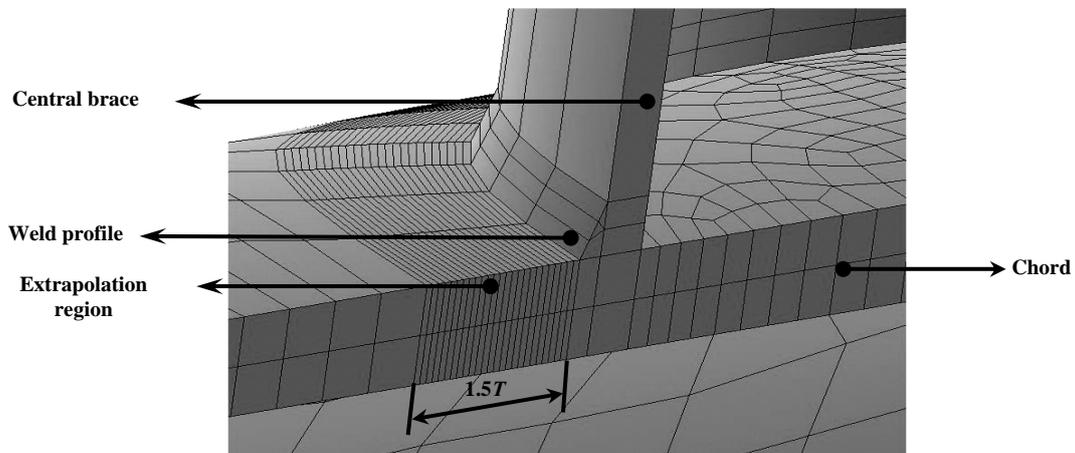
geometry. The location from which the stresses have to be extrapolated, extrapolation region, depends on the dimensions of the joint and on the position along the intersection. According to recommendations of IIW-XV-E [27], the first extrapolation point should be at a distance of  $0.4T$  from weld toe, and the second point must be  $1.0T$  further from the first point (Figure 6a). The HSS is obtained by linear extrapolation of the geometric stresses at these two points to the weld toe. At an arbitrary node inside the extrapolation region, the stress component in the direction perpendicular to the weld toe can be calculated, through the transformation of primary stresses in the global coordinate system, using the following equation:

$$\sigma_{\perp N} = \sigma_x l_1^2 + \sigma_y m_1^2 + \sigma_z n_1^2 + 2(\tau_{xy} l_1 m_1 + \tau_{yz} m_1 n_1 + \tau_{zx} n_1 l_1) \tag{5}$$

where  $\sigma_a$  and  $\tau_{ab}$  ( $a, b = x, y, z$ ) are components of the stress tensor which can be extracted from ANSYS analysis results; and  $l_1, m_1,$  and  $n_1$  are transformation components. At the saddle, crown  $0^\circ$ , and crown  $180^\circ$  positions, Eq. (5) is simplified as:



(a) Global view of the generated mesh



(b) Generated mesh in the regions adjacent to the

**Figure 5. Mesh generated based on the sub-zone approach**

$$\sigma_{\perp N} = \sigma_y m_1^2 + \sigma_z n_1^2 + 2\tau_{yz} m_1 n_1 \text{ (Saddle);}$$

$$\sigma_{\perp N} = \sigma_x \text{ (crown } 0^\circ \text{ and crown } 180^\circ) \quad (6)$$

Transformation components can be obtained as follows:

$$m_1 = \cos(X_{\perp}, y) = (y_w - y_n) / \delta,$$

$$n_1 = \cos(X_{\perp}, z) = (z_w - z_n) / \delta \quad (7)$$

$$\delta = \sqrt{(x_w - x_n)^2 + (y_w - y_n)^2 + (z_w - z_n)^2} \quad (8)$$

where  $X_{\perp}$  is the direction perpendicular to the weld toe (Figure 6b);  $x$ ,  $y$ , and  $z$  are axes of the global coordinate system;  $(x_n, y_n, z_n)$  and  $(x_w, y_w, z_w)$  are coordinates of the considered node inside the extrapolation region and its corresponding node at the weld toe position, respectively; and  $\delta$  is the distance between the weld toe and the considered node inside the extrapolation region. The stress at an extrapolation point is obtained as follows:

$$\sigma_{\perp E} = \frac{\sigma_{\perp N1} - \sigma_{\perp N2}}{\delta_1 - \delta_2} (\Delta - \delta_2) + \sigma_{\perp N2} \quad (9)$$

where  $\sigma_{\perp Ni}$  ( $i = 1$  and  $2$ ) is the nodal stress in the immediate vicinity of the extrapolation point in a direction perpendicular to the weld toe (Eq.(6));  $\delta_i$  ( $i = 1$  and  $2$ ) is obtained by Eq.(8); and  $\Delta$  equals to  $0.4T$  and  $1.4T$  for the first and second extrapolation points, respectively (Figure 6b). The extrapolated stress at the weld toe position, HSS, is calculated by the following equation:

$$\sigma_{\perp W} = 1.4\sigma_{\perp E1} - 0.4\sigma_{\perp E2} \quad (10)$$

where  $\sigma_{\perp E1}$  and  $\sigma_{\perp E2}$  are the stresses at the first and second extrapolation points in the direction perpendicular to the weld toe, respectively (Eq.(9)).

If the considered nodes in the calculations of Eqs.(6)-(10) are located on the outer surface of the chord, the value of  $\sigma_{\perp W}$  obtained from Eq.(10) can be used as  $\sigma_o$  in Eq.(4); and if the considered nodes are located on the inner surface of the chord, the result of Eq.(10) is equivalent to  $\sigma_i$  which is required for the calculation of the DoB in Eq.(4).

To facilitate the calculation of DoB values, above formulation was implemented in a macro developed by the ANSYS parametric design language (APDL). The input data required to be provided by the user of the macro are the node number at the weld toe, the chord thickness, and the numbers of the nodes inside the extrapolation region. These nodes can be introduced using the graphic user interface (GUI).

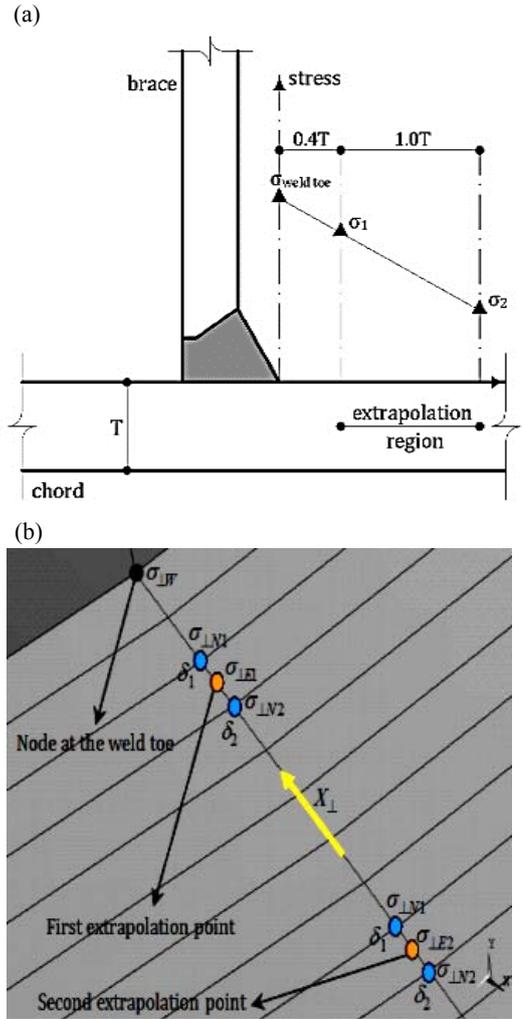


Figure 6. (a) Extrapolation method according to IIW-XV-E [27], (b) Interpolations and extrapolations required to extract the DoB based on the HSSs at the weld toe

### 3.5. Verification of the FE model

As far the authors can tell, there is no data available in the literature on the DoB values in KT-joints. However, a set of parametric equations have been developed by Morgan and Lee [3] for the prediction of DoB values in tubular K-joints under axial loading. These equations were used in the present study to validate the generated FE models.

In order to do so, an FE model was generated for a K-joint having typical geometrical characteristics (Table 1) and was analyzed under balanced axial loading.

Geometrical properties of the FE model were selected based on the data provided by HSE OTH 354 [28] for a steel specimen tested to determine the SCFs. The method of geometrical modeling (introducing the chord, braces, and weld profiles), the mesh generation procedure (including the selection of the element type and size), the analysis method, and the method of DoB extraction are identical for the validating model and the KT-joints used for the parametric study. Hence, the conclusion of the verification of the K-joint FE model with the results of equations proposed by Morgan and Lee [3] can be used to validate the generated KT-joint models. Results of verification

process have been presented in Table 2. It can be seen that there is a good agreement between the results of present FE model and equations proposed by Morgan and Lee [3]; and the maximum difference is less than 10%. Hence, generated FE models can be considered to be accurate enough to provide valid results.

**Table 1. Geometrical properties of the tubular K-joint specimen used for the verification of FE models**

Joint ID [28]	$D$ (mm)	$\tau$	$\beta$	$\gamma$	$\alpha$	$\theta$	$\zeta$
JISSP (3)	508	1	0.5	20.3	12.6	45°	0.15

**Table 2. Results of the verification of FE models based on available equations**

Position	DoB values		Difference
	Present FE model	Equations proposed by Morgan and Lee [3]	
Saddle	0.6494	0.5949	8.39%
Toe	0.8778	0.8983	2.33%
Heel	0.7602	0.6979	8.19%

#### 4. Investigation of the geometrical effects on DoB values

##### 4.1. Details of parametric investigation

In order to study DoB values in tubular KT-joints subjected to axial loading (Figure 1), 46 models were generated and analyzed using ANSYS. The objective was to study the effects of dimensionless geometrical parameters on the chord-side DoB values at the crown 0°, saddle, and crown 180° positions along the intersection between the chord and central brace.

Different values selected for parameters  $\beta$ ,  $\gamma$ ,  $\tau$ , and  $\theta$  have been presented in Table 3. These values cover the practical ranges of the non-dimensional parameters typically found in tubular joints of offshore jacket structures. Providing that the gap between the braces is not very large, the relative gap ( $\zeta = g/D$ ) has no considerable effect on the HSS values in a tubular K-joint. Hence, a typical value of  $\zeta = 0.3$  was designated for all joints.

**Table 3. Values selected for each non-dimensional parameter**

Parameter	Definition	Value(s)
$\beta$	$d/D$	0.4, 0.5, 0.6
$\gamma$	$D/2T$	12, 18, 24
$\tau$	$t/T$	0.4, 0.7, 1.0
$\theta$	-	30°, 45°, 60°
$\zeta$	$g/D$	0.3
$\alpha$	$2L/D$	16
$\alpha_B$	$2l/d$	8

Sufficiently long chord greater than six chord diameters (i.e.  $\alpha \geq 12$ ) should be used to ensure that the stresses at the brace/chord intersection are not affected by the chord's boundary conditions [8]. Hence, in this study, a realistic value of  $\alpha = 16$  was designated for all the models. It should be then noted that the results of present study are valid for  $\alpha \geq 12$ .

The brace length has no effect on HSSs when the parameter  $\alpha_B$  is greater than the critical value [11]. In the present study, in order to avoid the effect of short brace length, a realistic value of  $\alpha_B = 8$  was assigned for all joints. The 46 generated models span the following ranges of geometric parameters:

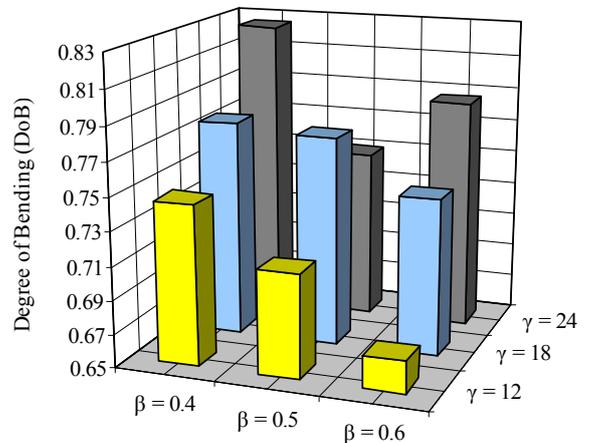
$$\begin{aligned}
 0.4 \leq \beta \leq 0.6 \\
 12 \leq \gamma \leq 24 \\
 0.4 \leq \tau \leq 1.0 \\
 30^\circ \leq \theta \leq 60^\circ
 \end{aligned} \tag{11}$$

##### 4.2. Effect of the $\beta$ on DoB values under axial loading

The parameter  $\beta$  is the ratio of brace diameter to chord diameter. Hence, the increase of the  $\beta$  in models having constant value of chord diameter results in the increase of brace diameter.

This section presents the results of investigating the effect of the  $\beta$  on the DoB. In this study, the influence of parameters  $\tau$ ,  $\gamma$ , and  $\theta$  over the effect of the  $\beta$  on the DoB was also investigated. For example, a chart is given in Figure 7 depicting the change of the DoB values at the saddle position due to the change in the value of the  $\beta$  and the interaction of this parameter with the  $\gamma$ . A large number of comparative charts were used to study the effect of the  $\beta$  and only one of them is presented here for the sake of brevity.

Results showed that the increase of the  $\beta$  generally results in the decrease of DoB values at the saddle position. Except from some irregularities detected in the change of the DoB in joints having big values of  $\gamma$  (e.g.  $\gamma = 24$ ), this conclusion is not dependent on values of other geometrical parameters.



**Figure 7. The effect of the  $\beta$  on DoB values at the saddle position ( $\tau = 0.7$ ,  $\theta = 45^\circ$ )**

### 4.3. Effect of the $\tau$ on DoB values under axial loading

The parameter  $\tau$  is the ratio of brace thickness to chord thickness and the  $\gamma$  is the ratio of radius to thickness of the chord. Hence, the increase of the  $\tau$  in models having constant value of the  $\gamma$  results in the increase of the brace thickness. Results of investigating the effect of the  $\tau$  on the DoB values are discussed in the present section. In this study, the interaction of the  $\tau$  with the other geometrical parameters was also investigated. For example, Figure 8 shows the change of the DoB values at the crown 0° and crown 180° positions due to the change in the value of the  $\tau$  and the interaction of this parameter with the  $\theta$ . Results indicated that the increase of the  $\tau$  generally leads to the increase of DoB values at both crown 0° and crown 180° positions. This behavior does not depend on values of other geometrical parameters.

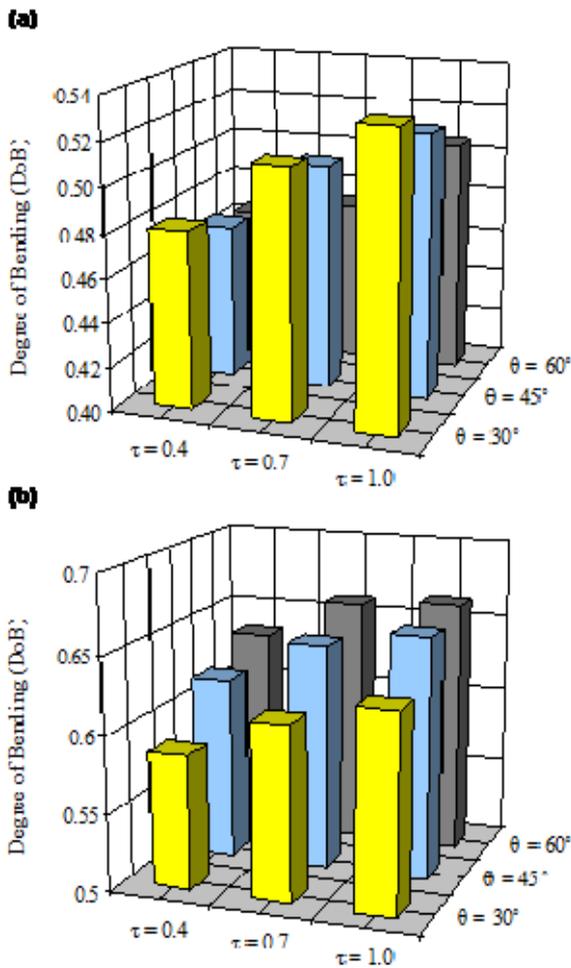


Figure 8. The effect of the  $\tau$  on DoB values:  
 (a) Crown 0° position ( $\beta = 0.4, \gamma = 12$ ),  
 (b) Crown 180° position ( $\beta = 0.6, \gamma = 24$ )

### 4.4. Effect of the $\theta$ on DoB values under axial loading

This section presents the results of studying the effect of the brace inclination angle,  $\theta$ , on DoB values and its interaction with the other geometrical parameters.

Three charts are given in Figure 9, as an example, depicting the change of the DoB at the crown 0°, saddle, and crown 180° positions due to the change in the value of  $\theta$  and the interaction of this parameter with the  $\beta$ .

Through investigating the effect of the  $\theta$  on DoB values, it can be concluded that the increase of the  $\theta$  leads to the decrease of the DoB at the saddle and crown 0° positions. On the contrary, at the crown 180° position, the increase of the  $\theta$  results in the increase of the DoB.

### 4.5. Effect of the $\gamma$ on DoB values under axial loading

The parameter  $\gamma$  is the ratio of radius to thickness of the chord. Hence, the increase of the  $\gamma$  in models having constant value of the chord diameter means the decrease of chord thickness. This section presents the results of investigating the effect of the  $\gamma$  on DoB values. In this study, the influence of parameters  $\beta, \tau,$  and  $\theta$  over the effect of the  $\gamma$  on DoB values was also investigated.

For example, three charts are presented in Figure 10 depicting the change of the DoB at the crown 0°, saddle, and crown 180° positions due to the change in the value of the  $\gamma$  and the interaction of this parameter with the  $\tau$ .

It was observed that the increase of the  $\gamma$  leads to the decrease of the DoB at the crown 0° and crown 180° positions. On the contrary, at the saddle position, the increase of the  $\gamma$  results in the increase of the DoB.

## 5. Development of the DoB parametric equations

The extensive use of the FE method, for the DoB analysis of tubular joints, is not feasible in a normal day-to-day design office operation. Instead, parametric design equations expressed in the form of dimensionless geometrical parameters are useful and desirable for fatigue design. Three parametric equations were developed in the present research for the calculation of DoB values at the crown 0°, saddle, and crown 180° positions in tubular KT-joints subjected to axial loading (Figure 1).

Parametric DoB equations were derived based on multiple nonlinear regression analyses performed by SPSS.

Values of dependent variable (i.e. DoB) and independent variables (i.e.  $\beta, \gamma, \tau,$  and  $\theta$ ) constitute the input data imported in the form of a matrix. Each row of this matrix involves the information about the DoB value at a considered position in a tubular KT-joint having specific geometrical properties. When the dependent and independent variables are defined, a model expression must be built with defined parameters. Parameters of the model expression are unknown coefficients and exponents.

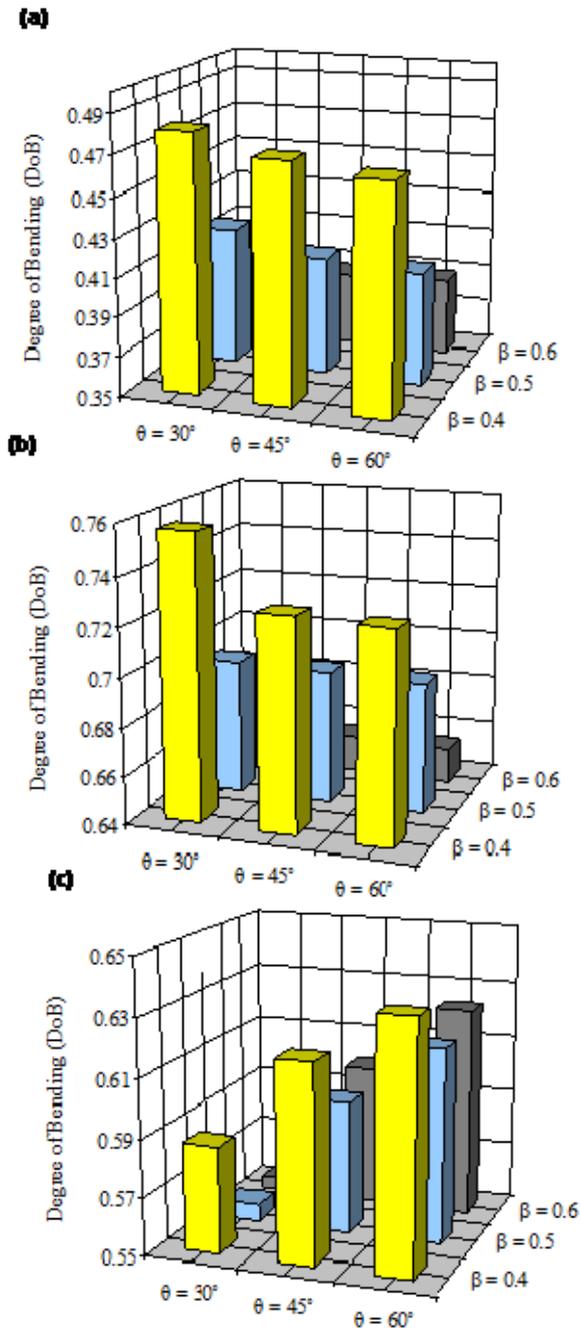


Figure 9. The effect of the  $\theta$  on DoB values ( $\tau = 0.4, \gamma = 12$ ): (a) Crown 0° position, (b) Saddle position, (c) Crown 180° position

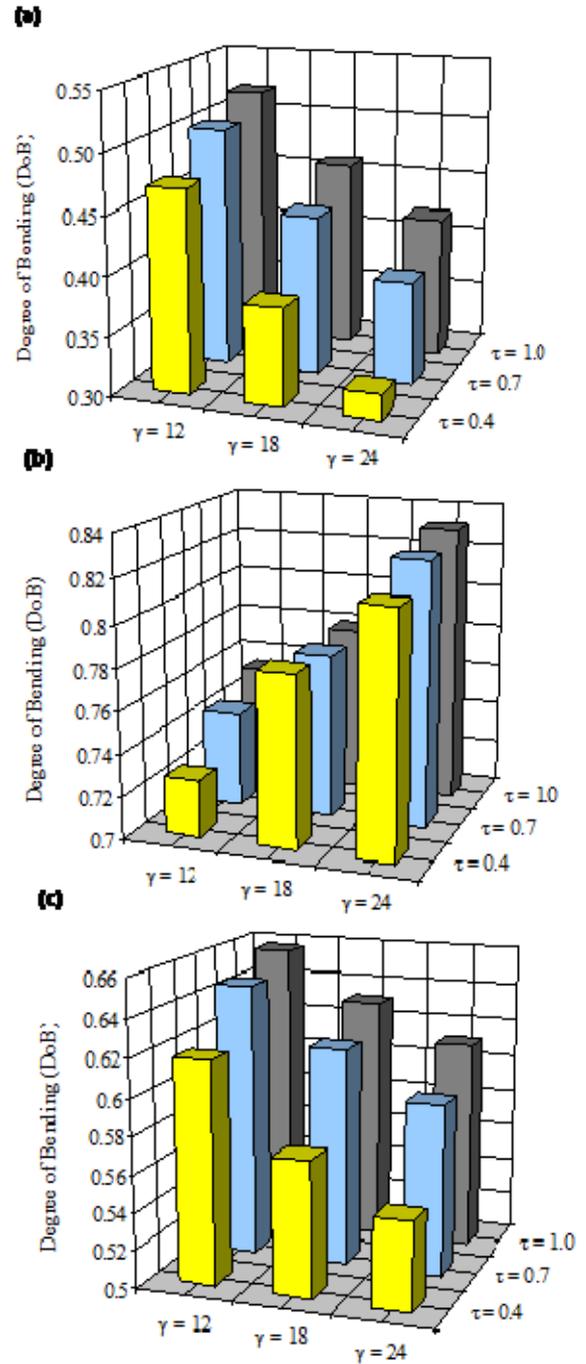


Figure 10. The effect of the  $\gamma$  on the DoB values ( $\beta = 0.4, \theta = 45^\circ$ ): (a) Crown 0° position, (b) Saddle position, (c) Crown 180° position

The researcher must specify a starting value for each parameter, preferably as close as possible to the expected final solution. Poor starting values can result in failure to converge or in convergence on a solution that is local (rather than global) or is physically impossible. Various model expressions must be built to derive a parametric equation having a high coefficient of determination ( $R^2$ ).

After performing a large number of nonlinear analyses, following parametric equations are proposed for the calculation of chord-side DoB at the crown 0°, saddle, and crown 180° positions in tubular KT-joints subjected to axial loading (Figure 1):

• **Crown 0°:**

$$\begin{aligned}
 \text{DoB} = & \left( (1 - 1.505\tau + 3.314\beta + 6.719\tau\beta) + \right. \\
 & \left. \left( (0.88 + 1.761\tau^{2.07}) \cdot (1 + 1.142\gamma^{3.454}) \cdot (0.002 - \right. \right. \\
 & \left. \left. 0.01\beta + 0.011\beta^2) \right) \right) \\
 & \cdot (1 - 0.3454\theta^{-0.062} - 0.105 \sin(\theta)) \\
 & \cdot (1 - 1.04\beta\tau^{0.115} + 1.14\gamma^{1.013} + \\
 & (0.001 \arcsin(\theta) \beta^{-4.201} - 1.205\gamma\tau^{-0.002} )
 \end{aligned}$$

$$R^2 = 0.989 \tag{12}$$

• **Saddle:**

$$\text{DoB} = 0.157\tau^{-0.015}\beta^{-0.951}\gamma^{0.42}\arcsin(\theta)^{-0.003} \\ \cdot (1 + 0.155\beta\tau^{0.477} - 0.443\gamma^{0.218} \\ + \beta \arcsin(\theta)^{-0.021})$$

$$R^2 = 0.957 \quad (13)$$

• **Crown 180°:**

$$\text{DoB} = 0.36\tau^{-0.202}\beta^{-0.508}\gamma^{0.881}\arcsin(\theta)^{0.01} \\ \cdot (1 + 1.763\tau^{0.3} + 0.5\gamma^{1.43} + \beta \arcsin(\theta)^{0.304} \\ - 0.31\tau\gamma^{0.041})$$

$$R^2 = 0.912 \quad (14)$$

In Eqs. (12)–(14), the parameter  $\theta$  should be inserted in radians. Obtained values of  $R^2$  are considered to be acceptable regarding the complex nature of the problem. Hence, proposed equations can reliably be used for the fatigue analysis and design of offshore jacket structures. The validity ranges of dimensionless geometrical parameters for the developed equations have been given in Eq. (11).

It should be noted that the inclination angles of the two outer braces may be different in practice. In such cases, it is suggested that the DoB should be calculated twice, based on the two different values of  $\theta$ ; and the DoB value that leads to a higher  $K$  (stress intensity factor) value and consequently lower number of cycles to failure should be selected to be used for fatigue analysis.

## 6. Conclusions

In the present paper, results of numerical study of DoB values in tubular KT-joints were presented and discussed. In this research program, a set of parametric stress analyses was performed on 46 KT-joint FE models subjected to axial loading. Analysis results were used to present general remarks on the effect of geometrical parameters including  $\tau$ ,  $\gamma$ ,  $\beta$ , and  $\theta$  on DoB values at the saddle, crown 0°, and crown 180° positions. Based on the verified FE results, a DoB database was prepared and a new set of DoB parametric equations was developed, based on nonlinear regression analyses.

Results showed that the increase of the  $\beta$  generally results in the decrease of DoB values at the saddle position. The increase of the  $\tau$  leads to the increase of DoB values at both crown 0° and crown 180° positions. The increase of the  $\theta$  results in the decrease of the DoB at the saddle and crown 0° positions. On the contrary, at the crown 180° position, the increase of the  $\theta$  leads to the increase of the DoB. The increase of the  $\gamma$  results in the decrease of the DoB at the crown 0° and crown 180° positions. On the contrary, at the

saddle position, the increase of the  $\gamma$  leads to the increase of the DoB.

Considering the relatively high coefficients of determination obtained, three proposed equations can reliably be used for the fatigue analysis and design of offshore jacket structures.

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