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Numerical Assessment of Tight-Fit Sleeve Clamp in the Repair Process of Cracked Submarine Pipelines

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ABSTRACT

The present research addresses the crack arrest in the submarine pipeline under internal pressure, axial force, and bending moment. The main purpose of the research is to consider tight-fit sleeves as a solution to crack arrest. The stress intensity factor criteria are used to describe the crack behavior. It should be noted that the cracks examined here are inclined through-thickness cracks, which the ABAQUS commercial software used to simulate them. It's noteworthy that Mode I fracture is dealt with, and the other fracture modes are omitted. The results show that the tight-fit sleeves, preferentially arrest the inclined cracks; so that the amounts of the stress intensity factors decrease for all the cracks except for the circumferential cracks to the extent that they become closed. As a result, their growth stops in practice; however, it best reduces the stress intensity factors by up to 65.36% at the circumferential cracks, and their amounts remain non-negative. Tight-fit sleeves create a pressurized region around the inclined cracks. This causes that inclined cracks remain closed.

1. Introduction

Submarine pipeline repair operations are usually costly and complex, so choosing an appropriate repair method can be efficient. In addition, if there is enough attention to repair the submarine pipeline, it will definitely provide a better service. Crack is always known as a defect in a pipeline system, even if it does not cause severe damage to the pipeline. The propagation of crack-type damage causes leakage and subsequent adverse environmental consequences, financial losses, and deferment of service, that's why turning to pipeline repair methods is crucial. In the current study, the necessity and importance of repair and crack arrest in submarine pipelines have been discussed.

The steel sleeve arrestors and composite crack arrestors are the most promising technologies for the repair of cracks [1]. The composite crack arrestors used for repair and reinforcement of pipes is intended as a permanent repair solution. Ghaffari et al. [2] investigated this repair technique for repairing longitudinal cracks in pipes under internal cyclic pressure. They used finite element analysis (FEA) and mixed-mode condition to investigate fatigue crack growth behavior of repaired pipes under cyclic loading. In their study, the influences of patch thickness on the life extension of fatigue crack growth and shape of crack-front for repaired pipes have also been considered.

Benyahia et al. [3] investigated the circumferential through-thickness cracks behavior in the repaired pipe with composite wrap under interior pressure using 3D finite element analysis. It should be noted that the SIF has been adopted as the fracture criterion in their research. The results of their work showed that the SIFs in the repaired pipe reduce significantly at the crack tip, which can extend the service life of the pipe.

Meriem-Benziane et al. [4] considered the efficiency of the repaired axial cracks (with patches) in pipelines subjected to interior pressure by comparing both the repaired and unrepaired state. In their research, the three-dimensional finite element method was used to calculate the SIFs at the crack tip. For evaluation of the pipeline repair, the failure assessment diagram was provided to gain the safety factor value. The results of their research demonstrated that the safety factor value depends on the length of the crack and the interior pressure parameters. Achour et al. [5] studied the behavior of the circumferential through-thickness cracks in the repaired pipe with composite wrap under the bending load using 3D finite element analysis. In their study, the stress intensity factor (SIF) has been applied to the fracture criterion. They also analyzed the influences of the geometrical and mechanical properties of the adhesive layer on the change of the SIF at the front of the crack. The findings of their research indicated that the SIFs in the repaired pipe decrease significantly at the tip of the flaw, which can increase the pipe service life.

Zarrinzadeh et al. [6] examined the inclined through-wall crack repaired with the composite patch in an aluminum pipe. They considered the behavior of fatigue cracks growth via empirical tests. In their research, SIFs have been calculated for the aluminum pipe under axial force, in which the XFEM with three-dimensional degenerated elements was applied for this case. Experimental and numerical results indicated that fatigue life is extended to the cracked pipe repaired with the composite patch.

Rashed et al. [7] investigated the axial load and bending moment capacity of a circumferential through-wall cracked pipeline repaired with a composite sleeve. They used the 3D finite element method, and also employed the failure assessment diagram for considering the repaired pipe behavior. The results showed that, for the considered range of crack angles and applied loads, the interaction of brittle and ductile failure states is insignificant.

In recent works, researchers only focused on the composite crack arrestors; therefore, in this research, it has been attempted to investigate the mechanism of cracked pipelines repair with steel sleeve arrestors. It should be noted that among the steel sleeve arrestors, the tight-fit sleeve has been considered for this issue. The difference between this work with other previous works is on this, and it can be said that the innovation of the present work is considered. The main objective of the current research is to consider the tight-fit sleeves as a solution to crack arrest. What is important here is that the geometric characterizations of tight-fit sleeves can be very useful in reinforcing the damaged pipeline segment.

2. Stress Intensity Factors for Stationary Cracks

It is noteworthy that the central hypothesis used all over in the present study is the theory of linear elastic fracture mechanics (LEFM). In the LEFM, the criteria of stress intensity factors (SIFs) are used to describe the crack behavior so the results of this article are only true for crack length below the critical crack length. Stress intensity factors are applied to express the stress field at the tip of the crack and are used as a severity quantity of crack tip for various configurations of the crack tip [8]. They play an essential role in the evaluation of cracks, which can be related to the critical stress levels caused by crack growth.

The modes of crack opening (i.e., Mode I, Mode II, and Mode III) have been presented in Figure 1. The theoretical explanation of the stress field about the tip of the crack is as follows:

$$\sigma = \frac{K_{\mathrm{I}}}{\sqrt{2\pi r}} f_I(\theta) + \frac{K_{\mathrm{II}}}{\sqrt{2\pi r}} f_{\mathrm{II}}(\theta) + \frac{K_{\mathrm{III}}}{\sqrt{2\pi r}} f_{III}(\theta) + \sigma_0 \quad (1)$$

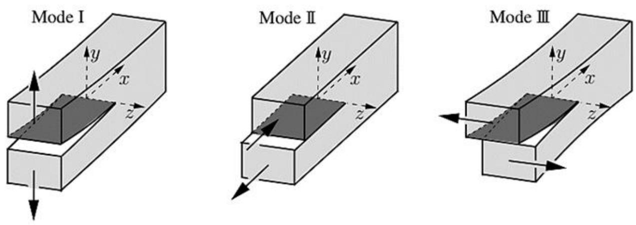

Figure 1. Crack opening modes [9]**.**

Figure 2 shows a schematic description of the angle, stress field, and radial distance. Equation (1) indicates that the stress field around the crack front has a singularity of the type $1/\sqrt{r}$. Thus, according to the assumptions of linear elastic fracture mechanics for a fictitious crack, $r \to 0$ is equivalent to $\sigma_{yy} \to \infty$.

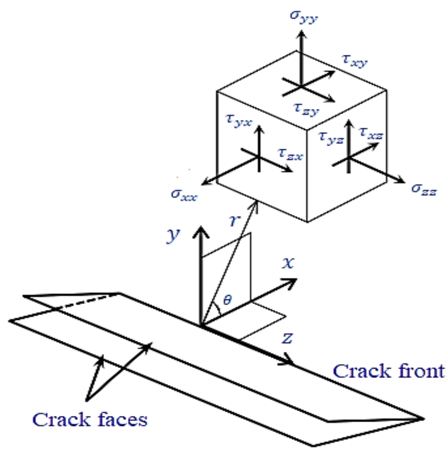

Figure 2. Cartesian 3D coordinate system for stresses around the crack front.

The severity of the stress field around the crack front is characterized by stress intensity factors. The stress intensity factors for all three modes are defined as follows:

$$K_{\mathrm{I}} = \lim_{r\to 0} \sqrt{2\pi r}\sigma_{yy}(r, \theta = 0) \quad (2)$$

$$K_{\mathrm{II}} = \lim_{r\to 0} \sqrt{2\pi r}\tau_{yx}(r, \theta = 0) \quad (3)$$

$$K_{III} = \lim_{r\to 0} \sqrt{2\pi r}\tau_{yz}(r, \theta = 0) \quad (4)$$

The two methods exist that can be applied to measure SIFs: one based on displacement and the other based on energy (i.e., J-integral). Here, the J-integral energy-based approach is used to determine the SIFs for stationary cracks, in which the interaction integral methods seem to be the most appropriate approach for separating stress intensity factors [10].
In 3D problems, the J-integral can be described as follows [11].

$$J = \lim_{\Gamma\to 0} \oint \left(\kappa\delta_{1i} - \sigma_{ij}\frac{\partial u_j}{\partial x}\right) d\Gamma \quad (5)$$

The J-integral for linear elastic fracture mechanics equals to strain energy release rate that can be expressed as follows [12].

$$J = \frac{K_I^2}{\bar{E}} + \frac{K_{II}^2}{\bar{E}} + \frac{(1+\nu)K_{III}^2}{E} \quad (6)$$

Where

$$\bar{E} = \begin{cases} E & \text{for plane stress condition} \\ \dfrac{E}{1-\nu^2} & \text{for plane strain condition} \end{cases} \quad (7)$$

To separate the fracture modes, the interaction integral approach as an accurate approach is employed [10]. In accordance with the interaction integral approach, the stress intensity factors can be described as follows [13].

$$\begin{Bmatrix} K_I \\ K_{II} \\ K_{III} \end{Bmatrix} = 4\pi \begin{Bmatrix} \dfrac{G}{4\pi} & 0 & 0 \\ 0 & \dfrac{G}{4\pi} & 0 \\ 0 & 0 & \dfrac{G}{4\pi(1-\nu)} \end{Bmatrix} \begin{Bmatrix} J_{int}^1 \\ J_{int}^2 \\ J_{int}^3 \end{Bmatrix} \quad (8)$$

Since the opening mode is the dominant mode, in this study, only this mode is considered, and the other fracture modes are disregarded.

3. Finite Element Modeling

In this study, ABAQUS commercial software [14] has been used to simulation and analysis of the cracked pipeline for both repaired and unrepaired states. The geometric characterizations of the model have been illustrated in detail in Figure 3.

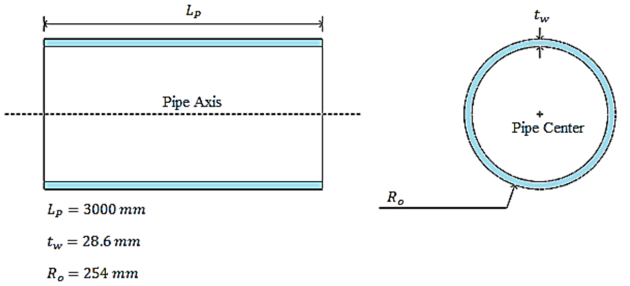

Figure 3. Geometric characterizations of the model.

The through-thickness cracks are assumed because this study only examines a state that the pipeline has leakage. Their geometry specifications, such as length and positioning angle relative to the circumferential direction of the pipeline, are presented in Table 1.

Table 1. Crack geometry specifications.

Description	Values
Inclined crack length (mm)	5, 25, 45, 65, 85 & 105
Crack inclination angle (deg)	0°, 15°, 30°, 45°, 60°, 75° & 90°

At the following, the tight-fit sleeve, that plays the role of reinforcement is introduced. The tight-fit sleeve creates a locking on the pipeline outer surface by contact interaction. Parameters such as contact length (L_C) , contact pressure, and friction coefficient are effective in locking capacity and play an important role in reducing axial stresses on the pipeline wall thickness. See Figure 4 for a better understanding.

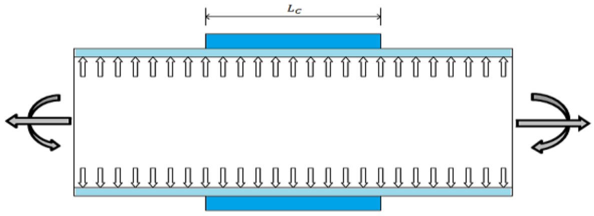

Figure 4. Tight-fit sleeve arrestor.

The most important part of modeling is meshing, which strongly affects the solution cost and accuracy of the solution. The crack or slit in the body produces a singularity. To consider the singularity effect, the singular elements are applied. For this, the crack tunnel around the front of the crack should be designed using the partition toolset. Within the crack tunnel, the quarter-point elements proposed by Barsoum [15] to produce a square-root singularity. It is noteworthy that the midpoints of these elements are situated in one-quarter of side length from the crack tip [16].
Due to the reduction of solution cost, the linear elements are applied for finite element analysis. The crack region with linear tetrahedron elements (4-node tetrahedron, C3D4) and others using linear hexahedron solid elements with reduced integration (8-node brick, C3D8R) are meshed, respectively. Tie constraints are

applied to the interfaces between regions with different elements. The finite element mesh for a cracked pipeline is shown in Figure 5. According to Figure 6, for modeling the tight-fit sleeve, the C3D8R elements are used.

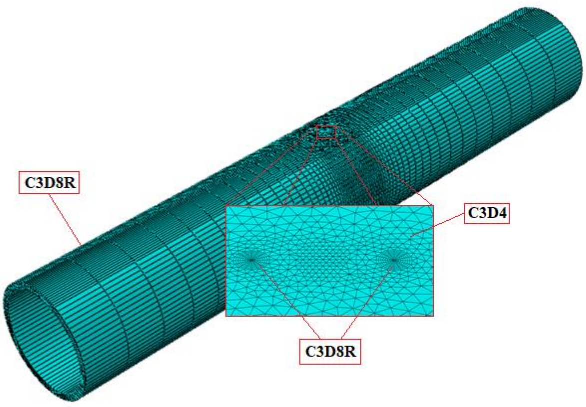

Figure 5. Finite element mesh around an inclined crack.

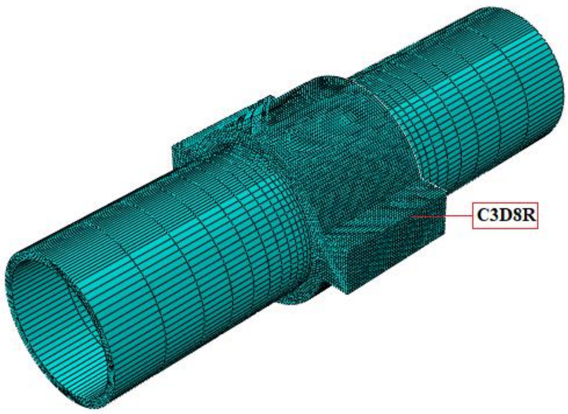

Figure 6. Finite element mesh for repaired pipeline.

The contact between the pipeline and tight-fit sleeve with contact pair (surface to surface contact) is modeled, and also, to define the friction between surfaces, the penalty method is applied. The static friction coefficients for the contact between surfaces of 0.3 has been considered here. In this study, three different contact lengths 330 mm, 440 mm, and 660 mm are considered.

Pipeline and tight-fit sleeve properties are presented in Table 2 and Table 3, respectively.

Table 2. Pipeline properties.

Description	Unit	Values
Pipeline material	–	API 5L-X60
Young's modulus	MPa	207,000
Poisson ratio	–	0.3
Yield stress	MPa	414
Ultimate stress	MPa	650
Density	kg/m^3	7,850

Table 3. Tight-fit sleeve properties.

Description	Unit	Values
Tight-fit sleeve material	–	ASTM A216
Young's modulus	MPa	210,000
Poisson ratio	–	0.3
Yield stress	MPa	220
Ultimate stress	MPa	500
Density	kg/m^3	7,820

It is noteworthy that the loads acting on the cracked pipeline have been extracted from operational conditions. The loads include internal pressure, tensile loading, and bending moment which are listed in Table 4. These are effective loads on the pipeline and can play a decisive role in pipeline behavior. It should be noted that these loads are obtained in operating conditions. Details of loading are shown schematically in Figure 7.

Table 4. Loads applied to the cracked pipeline.

Description	Unit	Values
Internal pressure	MPa	10
Axial force	N	1464511.723
Bending moment	$N.mm$	440019313.6

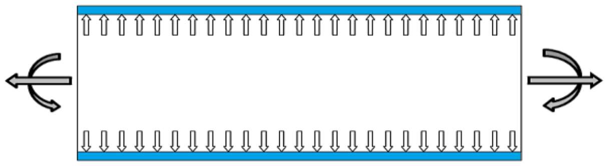

Figure 7. Pipeline under different loads.

The tension and bending moment with far-field stresses is applied to both ends of the model, and also the internal pressure is applied to the inner surface. Besides, the bolt loads are intended for the repaired state. In order to apply bending moment, reference node is used. As shown in Figure 8, symmetric force boundary condition is inserted at the both ends of pipe and in order to prevent the rigid body motion during analysis, at least one node must be fully restrained and it is important to know that the force bound

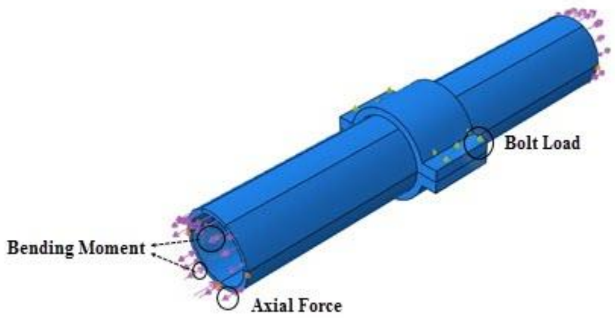

Figure 8. The loads applied to the repaired pipeline.

4. Verification of Numerical Solution

Here, numerical verification is done to obtain reliable results. As shown in Figure 9, a through-cracked cylindrical pressure vessel is considered which crack length, $2a$ and inclination angle, β are shown . For this, both analytical and numerical methods to determine the stress intensity factors are compared.

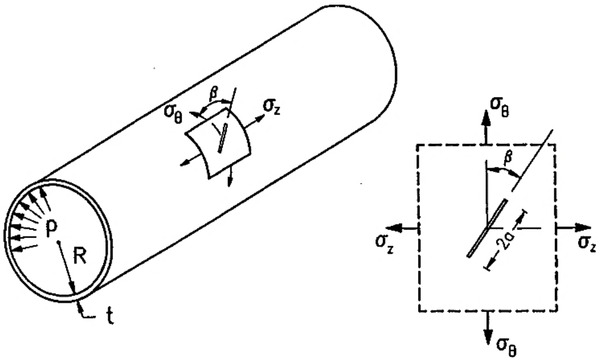

Figure 9. Through-cracked cylindrical pressure vessel [17].

The parameters specified in Figure 9 are as follows:

Table 5. The parameters specified according to Figure

Parameter	Description	Unit	Values
L	Length of vessel	mm	1000
R	Inner radius	mm	500
t	Thickness	mm	10
a	Crack half length	mm	10
β	Inclination angle	–	$\left[0^{rad}, \frac{\pi^{rad}}{2}\right]$
p	Internal pressure	MPa	1
σ_θ	Hoop stress	MPa	25
σ_z	Longitudinal stress	MPa	50

The values of stress intensity factors in Mode I is analytically determined by [17]

$$K_I = \frac{pR}{2t}\sqrt{\pi a}(1 + \sin^2 \beta) \tag{9}$$

Figure 10 illustrates the finite element modeling of a through-cracked cylindrical pressure vessel, in which several nodes are constrained to prevent the rigid body motion.

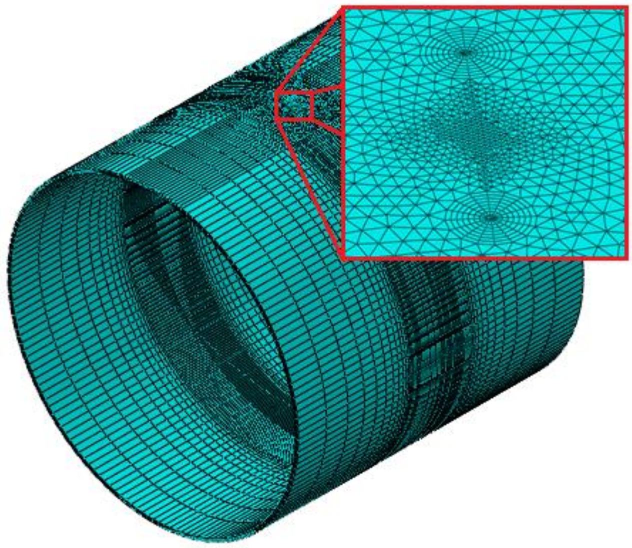

Figure 10. Finite element mesh for a through-cracked cylindrical pressure vessel.

Figure 11 indicates the stress intensity factor versus crack inclination angle for Mode I fracture. It is observed that a good agreement between the numerical and analytical results exist so that the maximum difference between them is less than 2%.

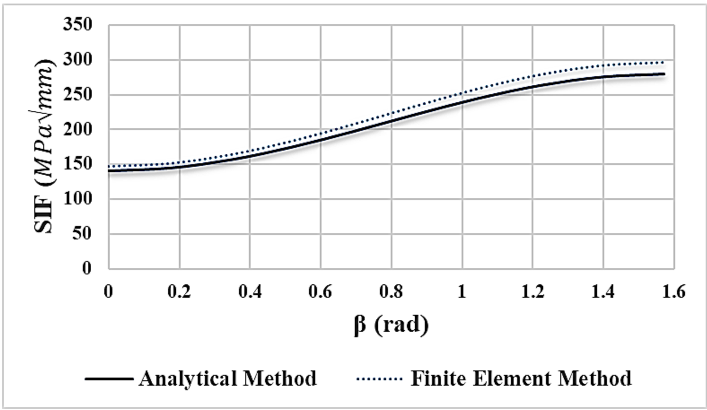

Figure 11. Comparison between numerical and analytical methods.

5. Results and Discussion

Here, the crack response behavior for the cracked pipeline in both unrepaired and repaired states is evaluated. It is worth noting that in the linear elastic fracture mechanics theory, stress intensity factors are used to describe crack behavior and by comparing the stress intensity factors for all three fracture modes, it is concluded that the opening mode is more dominant than the two transverse shear modes, therefore, in this study, only the opening mode is considered, and the shear modes are disregarded.

Figure 12 shows that with increasing relative crack length (i.e., crack length to pipeline thickness ratio), the stress intensity factors for fracture Mode I increase. Besides, Figure 13 indicates when the crack is changing from a circumferential direction to a longitudinal direction, the stress intensity factors are reduced. Circumferential cracks are completely subjected to axial stresses, whereas the longitudinal cracks are also mainly affected by hoop stresses (due to internal pressure). Other inclined through cracks are between circumferential and longitudinal cracks in terms of behavior.

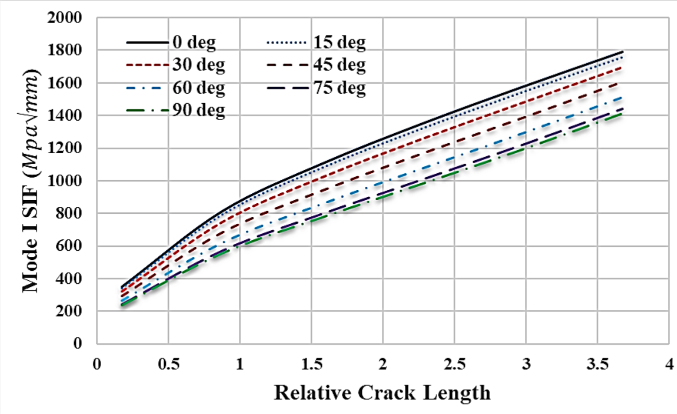

Figure 12. Mode I SIF vs. Relative Crack Length (Unrepaired state).

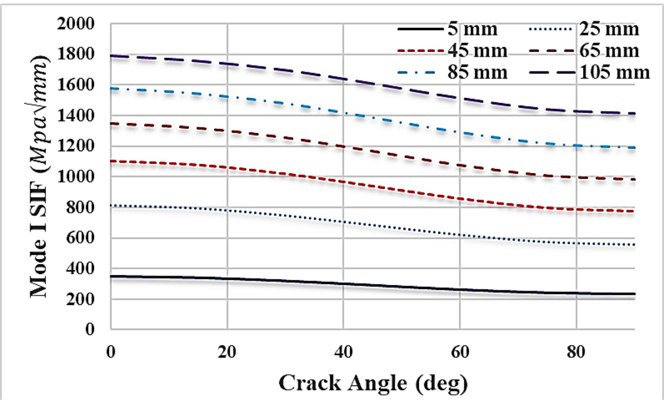

Figure 13. Mode I SIF vs. Crack Inclination Angle (Unrepaired state).

The response behavior of stationary cracks in the pipeline repaired with the tight-fit sleeve is also evaluated using stress intensity factor criteria. Figure 3 to 18 indicate the comparison between stress intensity factors for unrepaired pipeline and repaired pipeline with various contact lengths. At first glance, it can be said that this repair method significantly reduces the stress intensity factors, especially when the contact length is longer. Figure 14 illustrates that for various contact lengths 330 mm, 440 mm, and 660 mm, the stress intensity factors of the circumferential cracks decrease on average by 23.6%, 38.74%, and 65.36%, respectively. Figure 15 also indicates that for contact lengths 330 mm, 440 mm, and 660 mm, the stress intensity factors for inclined cracks with inclination angle 30° reduce on average by 70.65%, 84.64%, and more than 100%, respectively. Figure 16, Figure 17, and Figure 18 illustrate that the stress intensity factors for inclined cracks with inclination angles of 45°, 60°, and 90° reduce by more than 100% because they have negative values.

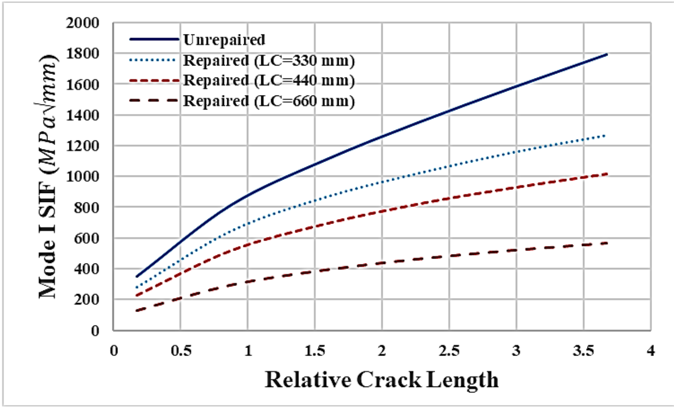

Figure 3. Crack response for the repaired pipeline (Circumferential cracks).

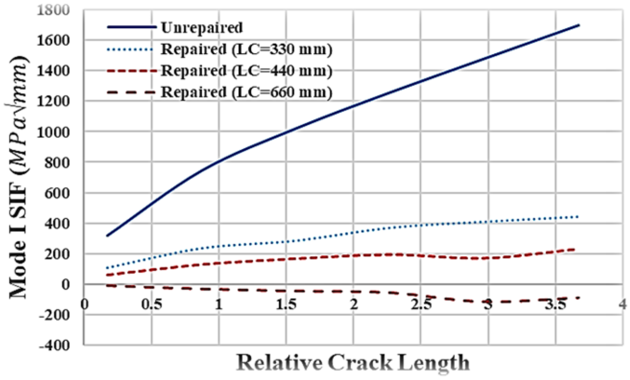

Figure 4. Crack response for the repaired pipeline (Inclined cracks with inclination angle 30°).

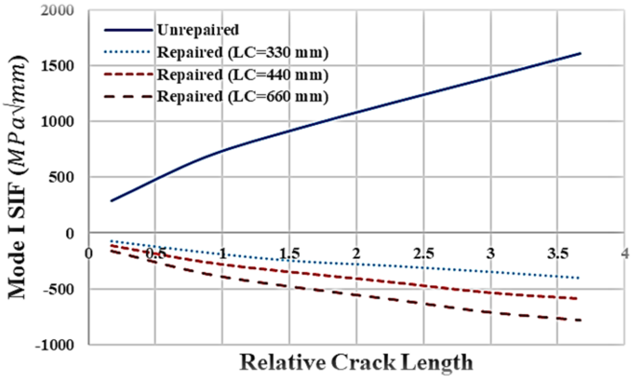

Figure 5. Crack response for the repaired pipeline (Inclined cracks with inclination angle 45°).

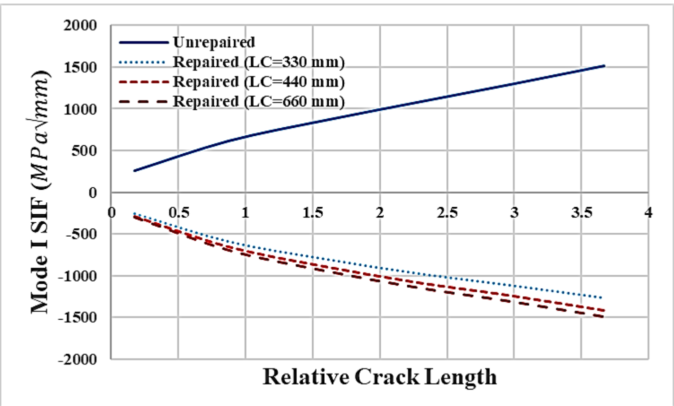

Figure 6. Crack response for the repaired pipeline (Inclined cracks with inclination angle 60°).

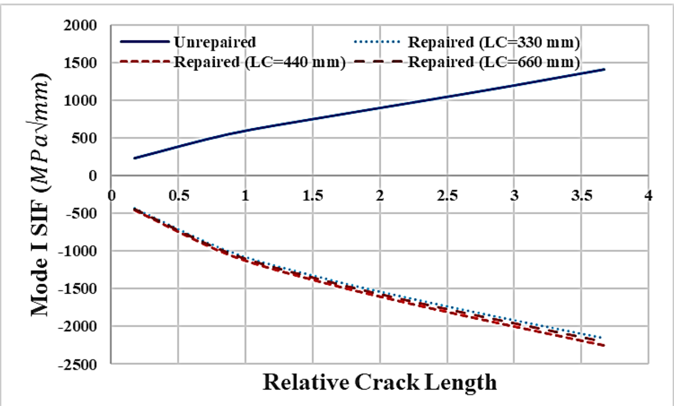

Figure 7. Crack response for the repaired pipeline (Longitudinal cracks).

As can be seen from the figures above, when the crack inclination angle changes from 0° to 90°, the stress intensity factors decrease further. This means that the inclined cracks near to longitudinal direction show an appropriate response, while the inclined cracks near to circumferential direction are not like that. It is worth noting that tight-fit sleeves have a better performance in the arrest of longitudinal cracks compared with other cracks because these types of clamps relieve hoop stresses to some extent that are applied compressively to the pipeline wall thickness. Pressure stresses cause the crack to fully arrest, and as a result, its propagation practically is stopped. One point to note here is that hoop stresses compared with axial stresses have a dominant performance in the arrest of all cracks except for circumferential cracks. The results indicate that axial and hoop stresses act as tensile and compressive stresses on the pipeline wall thickness, respectively. Increasing the contact length of the tight-fit sleeves, the pressure area around the crack tip is developed; hence negative stress intensity factors occur in opening mode. The results presented in Figure 15 confirm this so that in the inclined crack with inclination angle 30° with increasing contact length, stress intensity factors are reduced to the degree that results in negative values for these parameters. In inclined cracks with inclination angles higher than 30°, the stress intensity factor values are negative, so these cracks have been completely arrested. Circumferential cracks are also always affected by tensile stresses, so in these conditions, the stress intensity factors remain nonnegative. Still, the reduction of stress intensity factors for these cracks is significant.

It is necessary to mention that, in this article, it is intended to do investigations on the effects of some parameters and in order to observe their influences, it was necessary to compare them, so it was needed some numerical values to be reported to make comparisons possible. It must be emphasized that negative SIF in this article just used for comparison of results and this value does not have any physical meaning.

6. Conclusions

The present study addresses the crack arrest in submarine pipelines. The main purpose of the research is to consider the tight-fit sleeve as a solution to crack arrest. Some of the main conclusions of the conducted research are the following:

In this study, two factors play a significant role in the magnitude of crack driving forces (stress intensity factors) such as tensile stress and hoop stress. The tight-fit sleeves can reduce these stresses to an optimum level. In the tight-fit sleeves with increasing interaction contact length, the hoop stress is reduced, and consequently, the stress intensity factors decrease; however, the amount of decline for this parameter is not significant.

The results show that tight-fit sleeves favorably arrest inclined cracks so that values of stress intensity factors for all cracks except for circumferential cracks are reduced. And as a result, their growth stops; nevertheless, these reduce, at best, stress intensity factors in circumferential cracks by up to 65.36%, and their value remains non-negative.

In the pipeline repaired with tight-fit sleeves, as the crack length increases the amount of the stress intensity factors decreases further. Also, as the crack inclination angle increases, the amount of the stress intensity factors dramatically reduces.

In the tight-fit sleeves, it is better to use a lower contact length tight-fit sleeves to arrest the longitudinal crack because the results show that increasing interaction contact length does not significantly affect the response of the longitudinal cracks.

List of Symbols (Optional)

E	Young's modulus
f_α	The function of θ in the leading term, α mode
G	Shear modulus
J^i_{int}	Interaction integral, associated with an auxiliary Mode i
K_α	Stress intensity factor, α mode
L_C	Contact length

L_P	Segment pipe length
n_i	Unit normal vector
R_O	Outer radius
r	Polar coordinate, radius
t_W	Wall thickness
u_j	Displacement components
x, y, z	Cartesian 3D coordinate
Γ	Contour containing the crack tip
δ_{1i}	Kronecker delta
κ	Elastic strain energy density
ν	Poisson's ratio
σ	Cauchy stress
σ_{ij}	Stress tensor
σ_0	Finite stress
θ	Polar coordinate, angle

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